Key

EXAM 1 TMath 124 Winter 2021

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. [8] Find the limits (either numerically, graphically, or algebraically) if they exist:

start +.5
got it +.5
notation +.5
limit idea +.5

(a) $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

numerically:

x	$x^2\cos\left(\frac{1}{x^2}\right)$
-1	.5403
-.1	.008623
-.01	-.0000952
.1	.008623
1	.5403

So limit is zero

table +.5
values in table +.5
both sides +1

graphically: graph +.5, vis. +1, shape +.5

as $x \to 0$, $y \to 0$ so zero

algebraically:
note cosine returns values between -1 and 1 so +.5

$-1 \le \cos(1/x^2) \le 1$

Since $x^2 > 0$ we can mult above by x^2 to get

$-x^2 \le x^2\cos(1/x^2) \le x^2$ +.5

Apply lim as $x\to 0$ throughout then $\lim_{x\to 0} -x^2 = 0 = \lim_{x\to 0} x^2$ +.5

by the squeeze thm: zero +.5

(b) $\lim_{h\to 0} \dfrac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h}$

start +.5
got it +.5
notation +.5
limit idea +.5

algebraically:

$\lim_{h\to 0} \left(\frac{1}{(3+h)^2} - \frac{1}{9}\right) \div h$

$= \lim_{h\to 0} \frac{9-(3+h)^2}{9(3+h)^2} \div \frac{h}{1}$

$= \lim_{h\to 0} \frac{9-(9+6h+h^2)}{9(3+h)^2} \cdot \frac{1}{h}$

common den +.5
dist. negative +.5
divide by h +.5

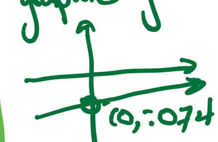
$= \lim_{h\to 0} \frac{-6h-h^2}{9h(3+h)^2}$

$= \lim_{h\to 0} \frac{\cancel{h}(-6-h)}{9\cancel{h}(3+h)^2}$

$= \lim_{h\to 0} \frac{-6-h}{9(3+h)^2} = \frac{-6-0}{9(3+0)^2} = \frac{-6}{9\cdot 9} = \frac{-2}{27} \approx -.074$

factoring +.5

graphically:



graph +.5

2. [5] Draw a graph for a function $\alpha(x)$, that satisfies all of the following: +1

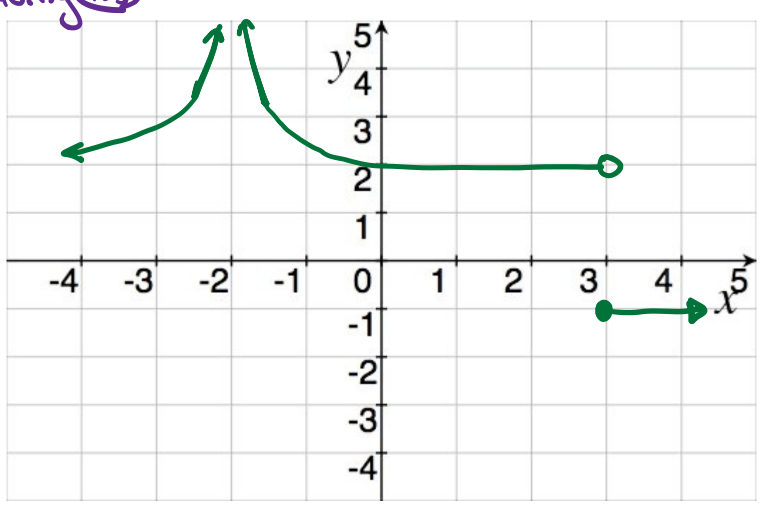
+1 (a) $\lim_{x\to -2} \alpha(x) = \infty$,

+1 (b) α is continuous on the interval $(-2, 3)$,

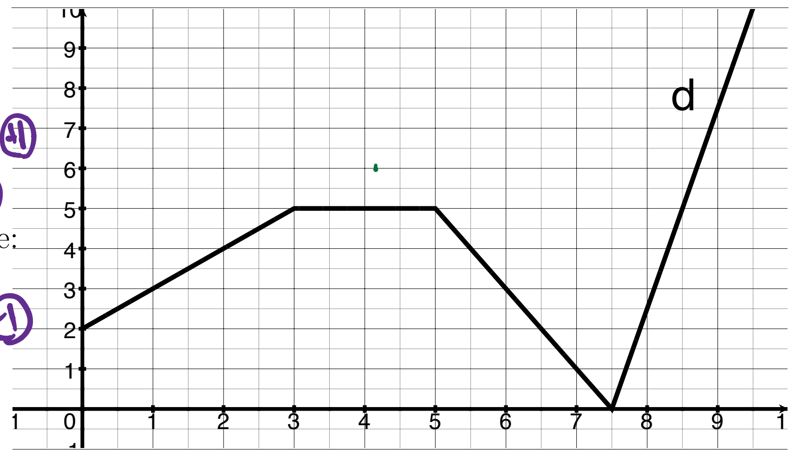
+1 (c) $\alpha(3) = -1$, and

+1 (d) $\lim_{x\to 3^-} \alpha(x) = 2$.

note, there are many correct answers!



3. The following graph is a function, d, that returns the distance (in feet) a fly is from a spider web after t seconds.



read graph +.5

(a) [2] How close does the fly get to the web and when?

touches the web (distance=0) +1

@ $t = 7.5$ seconds +1

(b) [5] Estimate the following, if possible:

$\lim_{x\to 3}(2d(\cancel{x}\,t) - 4)$ $= 2\lim_{x\to 3} d(x) - 4$ +1

$= 2\cdot 5 - 4 = 6$ +.5 +.5

$d(6)$

y value when $t=6$ ... 3 feet +1

$\frac{d}{d\cancel{x}\,t}d|_{\cancel{x}\,t=6}$

the slope of line tangent to d @ $t=6$ = $\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$ +.5 +.5 +1

(c) [3] What is the speed of the fly when $t = 6$ and is the fly moving towards or away from the web?

speed of fly = $\frac{d}{dt}d|_{t=6}$ ie. the question above? +1

$= 2$ ft/sec +1 moving towards the web +1

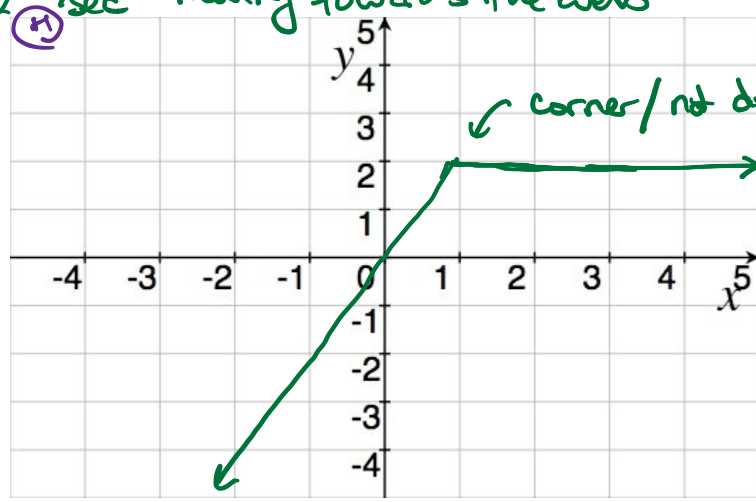
4. [5] Draw a graph for a function $\beta(x)$, that satisfies all of the following: +1

+1 (a) $\lim_{x\to\infty} \alpha(x) = 2$,

+1 (b) β is continuous on the interval $(-2, 3)$,

+1 (c) $\beta'(1)$ does not exist, and

+1 (d) $\beta'(x) > 0$ when $x < 0$.

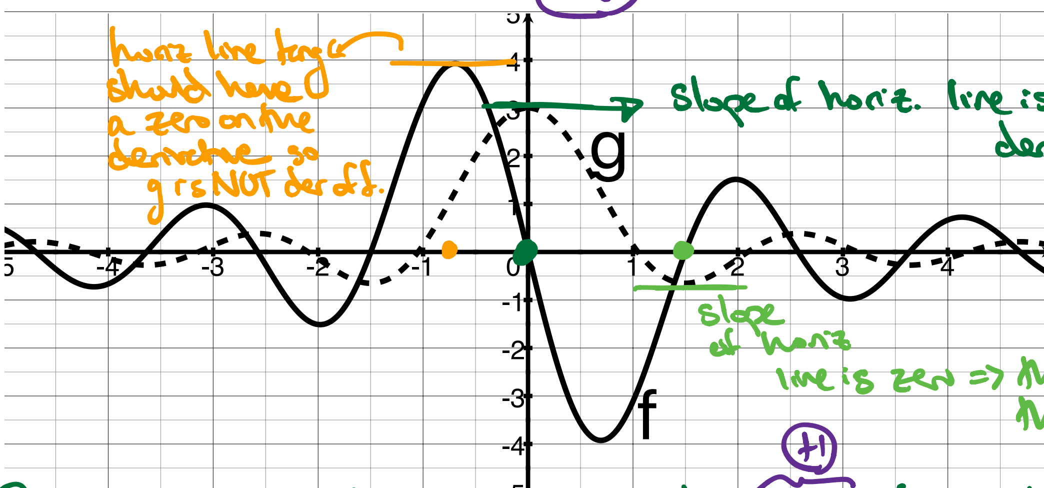


note, there are Many correct answers!

5. [3] Consider the graph of f and g shown below. One graph is the derivative of the other, that is, either $f'(x) = g(x)$ or $g'(x) = f(x)$. Determine which it is an explain/justify your choice!

+.5

start +.5
sense +1



horiz line tangent should have a zero on the derivative so g is NOT der of f.

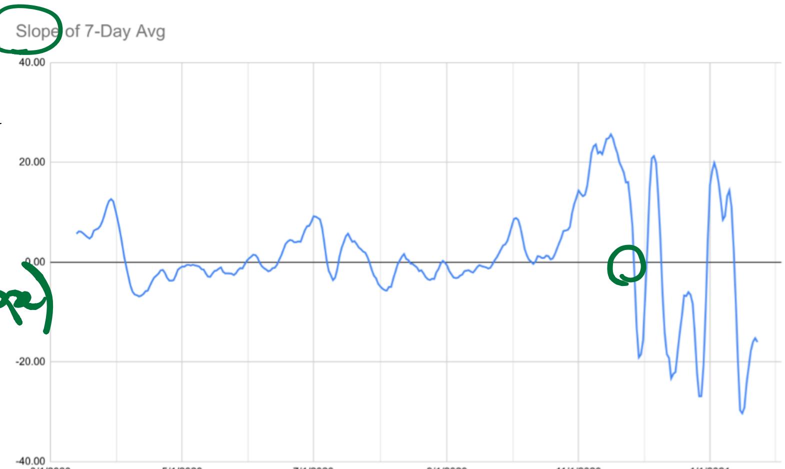
Slope of horiz. line is zero ⇒ the derivative passes thru x-axis.

slope of horiz line is zero ⇒ the derivative passes thru x-axis

+1

Recall the derivative is plotting the slope of the tangent line. Anytime the tangent line is horiz, there should be a zero/x-intercept on the graph of the derivative. Thus $g'(x) = f(x)$

6. Use the graph provided by JCRooks on CoronavirusWA Reddit on Jan 24th duplicated below. JCRooks is plotting the Slope of the 7-day average of new Covid-19 cases in King County over time.



(a) [2] Describe what is happening to the 7-day average of new Covid-19 cases in King County now.

+.5

sense +.5

The 7-day average is going down (corresponds with following a negative slope)

+1

(b) [3] Identify a time that the 7-day average of new Covid-19 cases peaked. Explain/justify your answer.

+.5

start +.5
got one +.5
explain +1.5

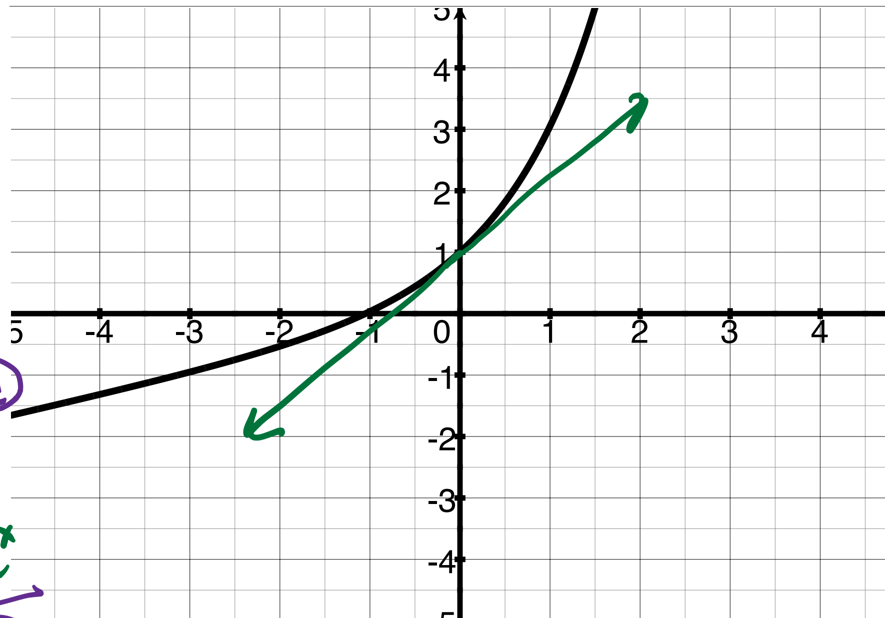
The circled time is a peak of the 7-day average. Before that time, the slope was positive so cases were increasing. After that time, the slope was negative so cases reduced ⇒ a maximal value.

(c) [2] Why do you think JCRooks provided this graph as opposed to the graph of Covid-19 cases directly?

7-day averages calms the data a bit more so it looks chaotic

Slopes are easier to compute

7. Consider $f(x) = \frac{1}{3}x + e^x$ graphed to the right.



(a) [3] Find $\frac{df}{dx}$

$\frac{df}{dx} = \frac{d}{dx}\left(\frac{1}{3}x + e^x\right)$

$= \frac{d}{dx}\left(\frac{1}{3}x\right) + \frac{d}{dx}(e^x)$ (+1)

$= \frac{1}{3}\frac{d}{dx}(x) + \frac{d}{dx}(e^x)$

$= \frac{1}{3} \cdot 1x^0 + e^x = \underbrace{\frac{1}{3}}_{(+.5)} + \underbrace{e^x}_{(+1)}$

(+.5) notation

(b) [1] Sketch the line tangent to f when $x = 0$.

tangent line (+.5) correct one (+.5)

(c) [4] Find the equation of the line sketched in part b. That is, find the equation of the line tangent to f when $x = 0$.

Looking for $y - y_1 = m(x - x_1)$ (+1)

m = slope of line tangent to f when x=0 (+1)

$= f'(0)$

$= \frac{1}{3} + e^0 = 4/3$ (+.5)

Passes thru $(0, f(0))$ from graph $(0,1)$ (+.5)

So

$y - 1 = 4/3(x - 0)$ (+1)

8. [4] For any problem on this exam,

(+2) (a) show a second way of approaching/building a solution, and

(+2) (b) explain why you did not choose this second method initially.

There are Lots of right answers here?