TMATH 124 MW: Quiz 4

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. Find $\frac{dy}{dx}$ given:

[2] (WebHW11 #3)
$y = 9\cos(x)\sin(y)$

$\frac{d}{dx}($
$\frac{dy}{dx} = 9\cos x \frac{d}{dx}(\sin(y)) + \frac{d}{dx}(9\cos x)\sin y$ product rule (+.5)
$\frac{dy}{dx} = 9\cos(x)\cos(y)\frac{dy}{dx} + 9(-\sin x)\sin y$ (+.5) derivatives (+.5)
$-9\cos(x)\cos(y)\frac{dy}{dx} - 9\cos(x)\cos(y)\frac{dy}{dx}$
$\frac{dy}{dx} - 9\cos(x)\cos(y)\frac{dy}{dx} = -9\sin(x)\sin y$
$\frac{dy}{dx}(1 - 9\cos(x)\cos(y)) = -9\sin(x)\sin(y)$
$\frac{dy}{dx} = \frac{-9\sin(x)\sin(y)}{1 - 9\cos(x)\cos(y)}$ (+.5) (+.5) rearrange

[2] (LogWks #2)
$y = [\log_2(x^3+2)^{51}]$

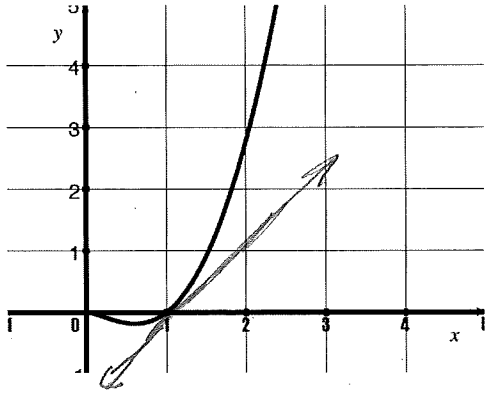
$y = 51\log_2(x^3+2)$
$f'(x) = x\frac{1}{\ln 2}$ (+.5) $f(x) = \log_2(x)$
$g'(x) = 3x^2$ $g(x) = x^3+2$ (+.5)
chain rule (+.5)
$\frac{dy}{dx} = 51\frac{1}{(x^3+2)\ln 2}\cdot 3x^2$
$= \frac{153x^2}{(x^3+2)\ln 2}$ (+.5)

[3] (§3.6 #42)
$y = \sqrt{x}e^{x^2-x}(x+1)^{\frac{2}{3}}$

into ln (+.5) ln rules (+1)
$\ln y = \ln x^{\frac{1}{2}} e^{x^2-x}(x+1)^{2/3}$
$\ln y = \ln x^{\frac{1}{2}} + \ln e^{x^2-x} + \ln(x+1)^{2/3}$
$\ln y = \frac{1}{2}\ln x + x^2 - x + \frac{2}{3}\ln(x+1)$
$\frac{d}{dx}($
$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\frac{1}{x} + 2x - 1 + \frac{2}{3}\frac{1}{x+1}(1)$ (+.5)
(+.5)
$\frac{dy}{dx} = y\left(\frac{1}{2x} + 2x - 1 + \frac{2}{3(x+1)}\right)$
solve for $\frac{dy}{dx}$ (+.5)

(2 product rules and 2 chain rules or)

2. (§3.6 #34) Consider $f(x) = x^2 \ln(x)$ whose graph is provided on the right.



(a) [1] Draw the equation of the line that is tangent to $f(x)$ when $x = 1$.

(b) [2] Find the equation of the line you drew in part a.

Looking for $y = mx + b$

(+.5) m = slope of line tangent to f @ $x = 1$

$= f'(1)$

$= 1 + 2(1)\ln(1)$

$= 1$

Finding $f'(x)$:

$f(x) = x^2 \ln x$

(+.5) product rule

$f'(x) = x^2 \frac{d}{dx}(\ln x) + \frac{d}{dx}(x^2)\ln x$

$f'(x) = x^2 \cdot \frac{1}{x} + 2x \cdot \ln x$

got it (+.5) $f'(x) = x + 2x\ln(x)$

passes thru $(1,0)$

So

$y - 0 = 1(x-1)$ or $0 = 1(1) + b$

$\Rightarrow -1 = b$

So $y = 1x - 1$

} got it (+.5)