Exam 1 # TMath 124 MW Winter 2016

Name: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

   T F $(x+y)^{-2} = \sqrt{x+y}$ $\quad (x+y)^{-2} = \frac{1}{(x+y)^2} \neq \sqrt{x+y} = (x+y)^{1/2}$

   T F $\lim_{x \to a} f(x) = f(\lim_{x \to a} x)$ $\quad$ only if f is continuous

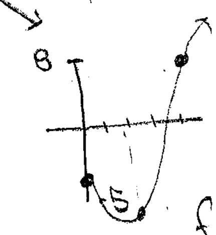
   T F If $f'(r)$ exists, then $\lim_{x \to r} f(x) = f(r)$.
   (derivative) (continuous)

   ← corner is a problem

   T F The absolute value function is a differentiable function.

   T F If f is continuous, $f(0) = -5$, and $f(4) = 8$, then $-5 \le f(2) \le 8$

   $f(2) < -5$

   T F If $\lim_{x \to a} g(x) = 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist.

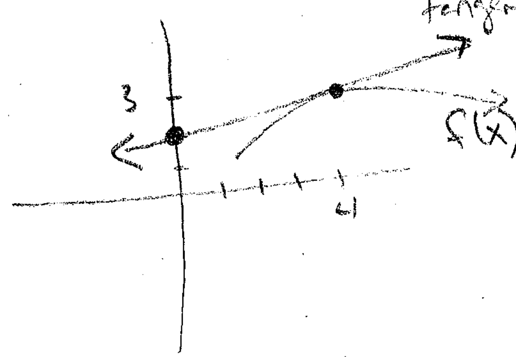
   let $g(x) = x-1$
   then $\lim_{x \to 1} g(x) = 0$ but $\lim_{x \to 1} \frac{(x-1)(x+2)}{x-1} = 3$

   T F $\lim_{x \to -1} (x^3 + 5x) = -6$

   $(-1)^3 + 5(-1) = -1 - 5$ b/c cont

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [2] (§2.7 #20) If the tangent line to $y = f(x)$ at $(4,3)$ passes through the point $(0,2)$ find the following.

   (a) $f(4)$ $\quad 3$

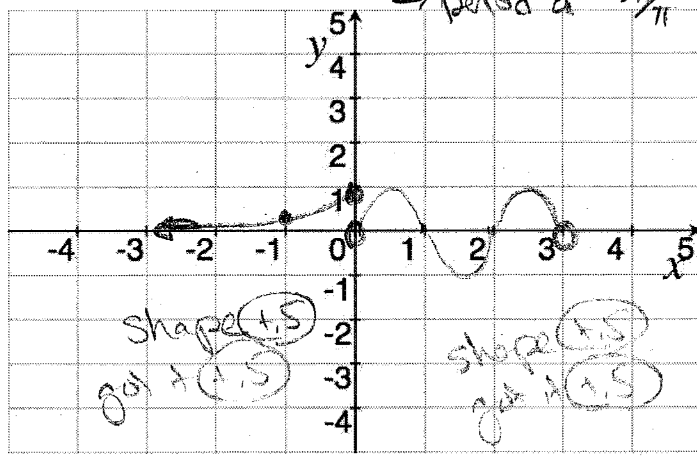
   (b) $f'(4)$ = slope of line tangent to f @ x = 4

   = $1/4$

3. Let f be a piece-wise defined function defined by $f(x) = \begin{cases} 3^x & \text{if } x \le 0, \\ \sin(\pi x) & \text{if } 0 < x < 3, \end{cases}$

$\hookrightarrow$ period of $2\pi/\pi$ or 2

(a) [2] (Quiz1 #1) Graph f on the axes provided.



(b) [1] (§2.2 #12) Determine the values of c for which $\lim_{x \to c} f(x)$ exists.

for all $x \neq 0$ and $x < 3$

— or —

$(-\infty, 0) \cup (0, 3)$

(c) [3] (WebHW3 #11) Evaluate the following (if they exist!)

$\lim_{x \to 3^-} f(x)$	$f(0)$	$\lim_{x \to 0^+} f(x)$
0	$3^0 = 1$	0

note: +.5 is 1

4. [4] Find the limit if it exists, or explain why it does not exist.

(InfLimitsWks #1)

notation (+.5)

$$\lim_{x \to \infty} \frac{x-2}{x^2-1}$$

(+.5)

$$\lim_{x \to \infty} \frac{x-2}{x^2-1} \cdot \frac{(1/x^2)}{(1/x^2)} = \lim_{x \to \infty} \frac{\frac{x}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$

alg (+.5)

$$\lim_{x \to \infty} \frac{\frac{1}{x} - \frac{2}{x^2} \to 0}{1 - \frac{1}{x^2}} = \frac{0}{1} = 0$$

(+.5)

or

leading x-m (+1)

$$\lim_{x \to \infty} \frac{x-2}{x^2-1} = \lim_{x \to \infty} \frac{x}{x^2}$$

Matthew's method

$$= \lim_{x \to \infty} \frac{1}{x} = 0$$

(+.5)

or

x	100	10,000	10,000,000
$\frac{x-2}{x^2-1}$			

got it (+.5) table (+.5) x values (+.5)

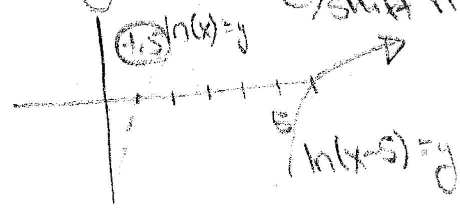
(PracticeExam #4)

notation (+.5)

$$\lim_{x \to 5^+} \ln(x-5)$$

graphically $\ln(x-5)$

$\hookrightarrow$ shift right 5 unit (+.5)



as $x \to 5^+$ $y \to -\infty$ (+.5)

or

numerically table (+.5)

x	5.1	5.0001	5.000001
$\ln(x-5)$			

x values (+.5) got it (+.5)

5. [4] Find the limit if it exists, or explain why it does not exist.

(§2.5 #36)

$\lim_{x \to \frac{pi}{2}} \sin(x + \cos(x))$

b/c sine & cosine are continuous and the composition of cont functions are continuous

Plug in (.5) eval trig (1) notation (.5)

$= \sin(\pi/2 + \cos(\pi/2))$

$= \sin(\pi/2 + 0)$

$= \sin \pi/2 = 1$

(§2.3 Lecture)

$\lim_{x \to 0} x^2 \sin \frac{\pi}{x}$

note DNE +.5 sense (+.5)

Algebraically: Notice $-1 \le \sin \pi/x \le 1$ (+.5)

$\Rightarrow -x^2 \le x^2 \sin \pi/x \le x^2$

Since $\lim_{x\to 0} -x^2 = 0 = \lim_{x \to 0} x^2$ (+.5)

by the squeeze thm $\lim_{x\to 0} x^2 \sin \pi/x$ (+.5)

or

Table (+.5)		
x	$-.0001$	$+.0001$
$x^2\sin(\pi/x)$		

2 sides (+1) notation (+.5)

6. [5] (ContWks #6) Sketch a graph of a function α that satisfies *all* of the following: (+1)

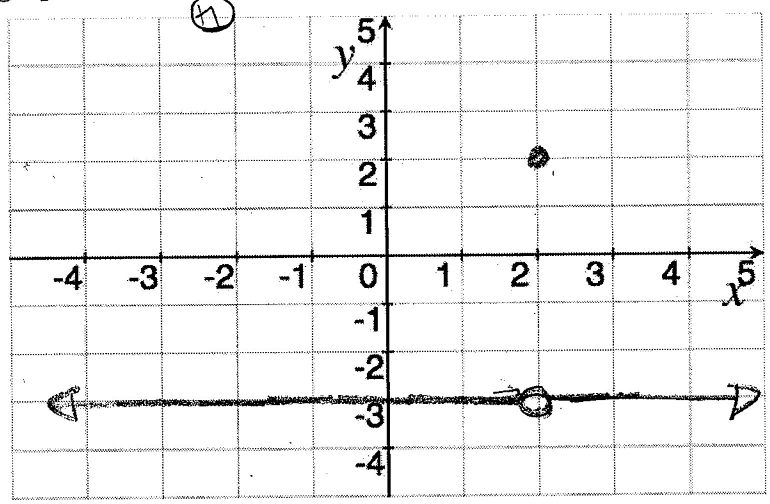
(a) $\alpha(2) = 2$ (+1)

(b) $\lim_{x \to 2} \alpha(x) = -3$ (+1)

(c) $\lim_{x \to \infty} \alpha(x) = -3$ (+1)

(d) α is continuous for $-4 \le x \le 1$ (+1)

Note: there are MANY answers

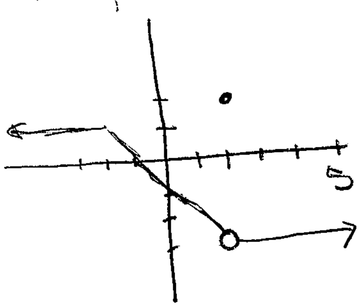


7. [3] Write the algebraic rule or the function α you created in the problem above.

$$\alpha(x) = \begin{cases} 2 & \text{if } x = 2 \\ -3 & \text{if } x \ne 2 \end{cases}$$

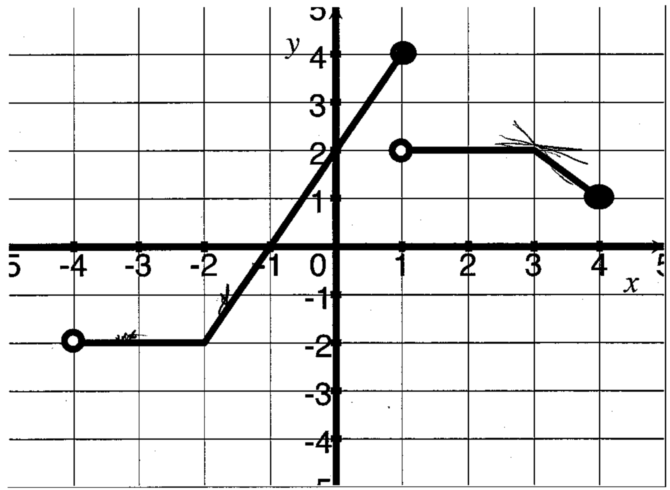
Start (+.5) function (+.5) match (+2)

Other possible answers



$$\alpha(x) = \begin{cases} 2 & \text{if } x = 2 \\ -3 & \text{if } 2 < x \\ -x-1 & \text{if } -2 < x < 2 \\ 1 & \text{if } x \le -2 \end{cases}$$

8. Consider the graph of the piece-wise defined function g to answer the following questions

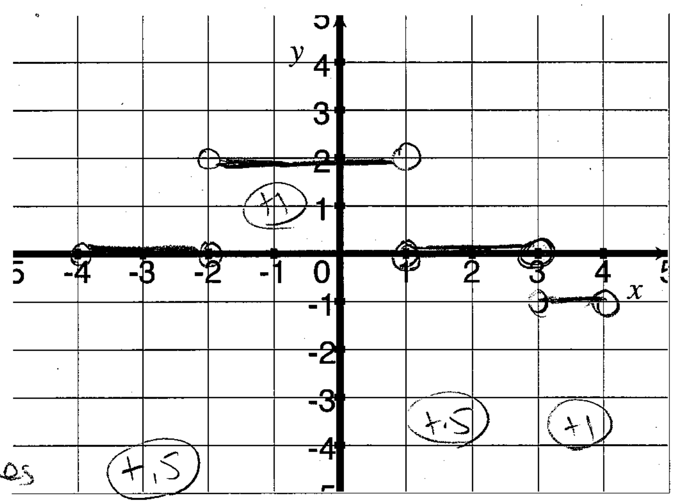


(a) [1] (WebHW2 #1)
$g(1)$ 4

(b) [1] (WebHW2 #1)
$\lim_{x \to 3} g(x)$ 2

(c) [1] (Quiz2 #3)
$g'(3)$ DNE

(d) [2] (Quiz2 #3)
$\frac{d}{dx} g|_{x=0}$ 2



(e) [4] (WebHW5 #6)
Draw a graph of $g'(x)$

endpoints (+.5)
slopes of tang lines (+.5)

9. (WebHW5 #3) [5] Let $f(x) = 4x - x^2$. Find the equation for the line tangent to the graph of f, when $x = 1$.

equation of a line: $y = mx + b$ (+.5)

m = slope of line tang. to f @ $x = 1$

$= f'(1)$ (+.5)

$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[4(1+h) - (1+h)^2] - [4(1) - 1^2]}{h}$ (+1)

$= \lim_{h \to 0} \frac{\cancel{4} + 4h - \cancel{1} - 2h - h^2 - \cancel{4} + \cancel{1}}{h} = \lim_{h \to 0} \frac{\cancel{h}(4 - 2 - h)}{\cancel{h}}$

$= \lim_{h \to 0} 2 - h = 2$ alg (+1) limit eval (+.5)

Passes thru the point $(1, f(1))$ or $(1, 3)$ (+.5)

So (+.5) $3 = 2(1) + b \Rightarrow b = 1$

Eq. of line: $y = 2x + 1$ (+.5) plug in values

or

equation of a line: $y = mx + b$ (+.5)

m = slope of line tangent to f @ $x = 1$

$= f'(1)$ (+.5)

notice $f'(x) = 4(1)x^0 - 2x$

$= 4 - 2x$ (+1)

So $f'(1) = 4 - 2 = 2$ (+1)

line passes thru $(1, 3)$ (+.5)

So $3 = 2(1) + b \Rightarrow b = 1$ (+.5)

Eq of line $\Rightarrow y = 2x + 1$ (+.5) note: I think this problem is not very accurate -- gravity on Mars is different.

10. If a rock is thrown upward on the planet Mars with a velocity of 8m/s, its height (in meters) after t seconds is given by $H(t) = 8t - 2t^2$.

   (a) [2] Find a function that describes the instantaneous velocity of the ball after t seconds.

   (b) [2] When does the ball reach its highest point?

   (c) [1] When does the rock hit the surface?

(a) average velocity $= \dfrac{\Delta H}{\Delta t}$

instantaneous velocity $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta H}{\Delta t}$ (+.5) the derivative?

$= \lim\limits_{h \to 0} \dfrac{H(t+h) - H(t)}{h}$

$= \lim\limits_{h \to 0} \dfrac{[8(t+h) - 2(t+h)^2] - [8t - 2t^2]}{h}$

alg (+.5)
limit (+.5)
notation (+.5)

$= \lim\limits_{h \to 0} \dfrac{\cancel{8t} + 8h - \cancel{2t^2} - 4th - 2h^2 - \cancel{8t} + \cancel{2t^2}}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(8 - 4t - 2h)}{\cancel{h}}$

$= 8 - 4t$

or

(+.5) the derivative? Nathan's Rule

$8t^1 - 2t^2$

$8(1)t^0 - 2(2)t^1$

$= 8 - 4t$

(+.5) (+.5)

notation (+.5)

(b) Reaches highest point @ the vertex

$8t - 2t^2$

or when inst. vel $= 0$ (+.5)

$8 - 4t = 0$ (+.5)

$-4t = -8$

$t = 2$ sec

alg (+.5) got it (+.5)

or

put parabola in vertex form

$-2t^2 + 8t = y$

$t^2 - 4t = -\frac{1}{2}y$

$t^2 - 4t + 4 = -\frac{1}{2}y + 4$

$(t-2)^2 = -\frac{1}{2}y + 4$

$(t-2)^2 - 4 = -\frac{1}{2}y$

$-2(t-2)^2 + 8 = y$ so vertex is $(2, 8)$

=> at 2 sec

(c) ie when is the height zero again?

(+.5) $0 = 8t - 2t^2$

$0 = t(8 - 2t)$

$t = 0$ or $8 - 2t = 0$

=> $t = 4$

so after 4 sec (+.5)