TMATH 124 UH: Quiz 4

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. Find $\frac{dy}{dx}$ given:

[2] (ImplicitWks #1b)
$y\cos(x) = y^2$

$\frac{d}{dx}$

product (+.5)

$y \frac{d}{dx}(\cos x) + \frac{d}{dx}(y)\cos(x) = 2y\frac{dy}{dx}$

$-y\sin x + \frac{dy}{dx}\cos(x) = 2y\frac{dy}{dx}$ (+.5)

$-\frac{dy}{dx}\cos(x) \qquad -\frac{dy}{dx}\cos x$

$-y\sin(x) = 2y\frac{dy}{dx} - \cos(x)\frac{dy}{dx}$

$-y\sin(x) = (2y - \cos(x))\frac{dy}{dx}$

$\frac{-y\sin(x)}{2y-\cos(x)} = \frac{dy}{dx}$

alg (+.5)

des of try (+.5)

or $\frac{y\cos(x)}{y} = \frac{y^2}{y}$

$\cos(x) = y$

$\frac{d}{dx}$

$-\sin(x) = \frac{dy}{dx}$

[2] (WebHW12 #6)
$y = \log_7(\sqrt{2x+7})$

$y = \log_7 (2x+7)^{1/2}$

$y = \frac{1}{2}\log_7(2x+7)$

Chain (+.5)

$f(x) = \log_7(x) \qquad f'(x) = \frac{1}{x\ln 7}$ (+.5)

$\underbrace{g(x) = 2x+7}_{(+.5)} \qquad g'(x) = 2$

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{(2x+7)\ln 7} \cdot 2$

$\left.\frac{dy}{dx} = \frac{1}{(2x+7)\ln 7}\right\}$ (+.5)

[3] (LogWks #1)
$y = x^{\sqrt{x}}$

into ln (+.5)

ln prop (+1)

$\ln y = \ln x^{x^{1/2}}$

$\ln y = x^{1/2}\ln x$

$\frac{d}{dx}$

product (+.5)

$\frac{1}{y}\frac{dy}{dx} = x^{1/2}\frac{d}{dx}(\ln x) + \frac{d}{dx}(x^{1/2})\ln x$

(+.5)

$\frac{1}{y}\frac{dy}{dx} = x^{1/2} \cdot \frac{1}{x} + \frac{1}{2}x^{-1/2}\ln x$

$\frac{dy}{dx} = y\left(\frac{x^{1/2}}{x} + \frac{\ln x}{2\sqrt{x}}\right)$

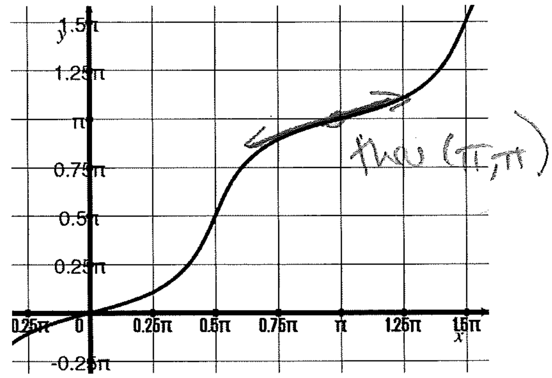
$\frac{dy}{dx} = y\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$ solve for dy/dx (+.5)

or $x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$

2. (§3.5 #26) Consider $\sin(x+y) = 2x - 2y$ whose graph is provided on the right.

(a) [1] Draw the equation of the line that is tangent to $f(x)$ when $x = \pi$.

(b) [2] Find the equation of the line you drew in part a.



Looking for $y = mx + b$

(.5) m = slope of line tangent to the graph @ $x = \pi$

$= \left.\frac{dy}{dx}\right|_{x=\pi}$

finding $\frac{dy}{dx}$:

$\sin(x+y) = 2x - 2y$

$\frac{d}{dx}($ chain (.5)

$\cos(x+y)\left(1 + \frac{dy}{dx}\right) = 2 - 2\frac{dy}{dx}$

$\cos(x+y) + \frac{dy}{dx}\cos(x+y) = 2 - 2\frac{dy}{dx}$

$-2 \qquad -2$

$\cos(x+y) - 2 = -2\frac{dy}{dx} - \frac{dy}{dx}\cos(x+y)$

$\cos(x+y) - 2 = \frac{dy}{dx}(-2 - \cos(x+y))$

$\frac{\cos(x+y) - 2}{-2 - \cos(x+y)} = \frac{dy}{dx}$

got it (.5)

$= \frac{\cos(\pi+\pi) - 2}{-2 - \cos(\pi+\pi)}$

$= \frac{\cos(2\pi) - 2}{-2 - \cos(2\pi)}$

$= \frac{1-2}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$

So

$y - \pi = \frac{1}{3}(x - \pi)$ or

using $y = mx + b$

$\pi = \frac{1}{3}\pi + b$

$\frac{2}{3}\pi = b$

got it (.5)

So $y = \frac{1}{3}x + \frac{2\pi}{3}$