Exam 2

TMath 124

MW Winter 2016

Name: Key

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

T (F) $\frac{d}{dt}x^3 = 3x^2\frac{dx}{dt}s$ → what???

T (F) $\lim_{x\to 0} x^2 = 2x$ $\lim_{x\to 0} x^2 = 0^2 = 0$

T (F) $(3^x)' = x \cdot 3^{x-1}$ $(3^x)' = 3^x \ln 3$

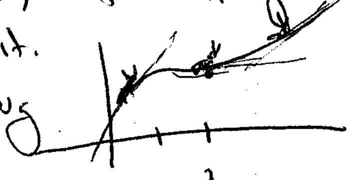
T (F) $x^3 = 3x^2$ $(x^3)' = 3x^2$ but we need a derivative

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [2] Explain what $f'(2)$ is as you would to a third grader.

slope/steepness (+.5)
tangent line / line of sight (+.5)
at the pt $x=2$ (+.5)
right idea (+.5)

Look at the graph of f & imagine a bug crawling along it. Notice that the bug has no neck so can only look in one direction (ignoring the fact that they have amazing eyes). The steepness of the bug's line of sight when ~~the~~ the bug is above $x=2$ is $f'(2)$.



$(x^2)^3 = x^2 x^2 x^2 = x^6$

3. [4] Find $\frac{dy}{dx}$ of the following:

(WebHW9 #5)

$y = 5e^x \cos(x)$

Product rule (+.5) notation (+.5)

$\frac{dy}{dx} = 5e^x \frac{d}{dx}(\cos(x)) + \frac{d}{dx}(5e^x)\cos(x)$

$= 5e^x(-\sin(x)) + 5e^x \cos(x)$ (+.5) (+.5)

$= -5e^x \sin(x) + 5e^x \cos(x)$

(§3.4 #26)

$y = \frac{(x+3)^4}{(x^2+3x)^5}$

quotient rule (+.5) correct quotient (+.5)

$\frac{dy}{dx} = \frac{(x^2+3x)^5 \frac{d}{dx}[(x+3)^4] - (x+3)^4 \frac{d}{dx}[(x^2+3x)^5]}{[(x^2+3x)^5]^2}$

$= \frac{(x^2+3x)^5 \cdot 4(x+3)^3(1) - (x+3)^4 \cdot 5(x^2+3x)^4(2x+3)}{(x^2+3x)^{10}}$ (+.5) (+1)

notation/got it (+.5)

or

$y = (x+3)^4 (x^2+3x)^{-5}$

product rule (+.5)
exp rules (+.5)

$\frac{dy}{dx} = (x+3)^4 \frac{d}{dx}[(x^2+3x)^{-5}] + \frac{d}{dx}[(x+3)^4](x^2+3x)^{-5}$

$= (x+3)^4(-5)(x^2+3x)^{-6}(2x+3) + 4(x+3)^3(x^2+3x)^{-5}$ (+1) (+.5)

notation/got it (+.5)

$y = \frac{(x+3)^4}{[x(x+3)]^5} = \frac{(x+3)^4}{x^5(x+3)^5} = \frac{1}{x^6+3x}$

$\frac{dy}{dx} = \frac{(x^6+3x)0 - 1(6x^5+3)}{(x^6+3x)^2}$

or using $y = (x^6+3x)^{-1}$ $\frac{dy}{dx} = -(x^6+3x)^{-2}(6x^5+3)$

4. [5] Find $\frac{dy}{dx}$ of the following:

(WebHW9 #8)

$y = 6^{15x}$

Chain (+.5)

$g(x) = 15x$ $g'(x) = 15$

$f(x) = 6^x$ $f'(x) = 6^x \ln 6$ (+.5)

$f(g(x)) = f(15x) = 6^{15x}$ (+.5)

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$= f'(15x) \cdot g'(x)$

$= 6^{15x} \ln 6 \cdot g'(x)$

$= 6^{15x}(\ln 6) \cdot 15$ got it (+.5)

(TrigWks #1)

$y = \sin(x)\sqrt{x^3 - 5}$

product rule (+.5) exp fraction (+.5)

$\frac{dy}{dx} = \sin(x) \frac{d}{dx}((x^3-5)^{1/2}) + \frac{d}{dx}(\sin x)(x^3-5)^{1/2}$

$= \sin(x) \frac{1}{2}(x^3-5)^{-1/2} \cdot 3x^2$ (+1) chain

$+ \cos(x)(x^3-5)^{1/2}$ (+.5)

got it/notation (+.5)

5. [4] Find the limit if it exists, or explain why it does not exist.

(TrigWks #2)

$$\lim_{x\to 0} \frac{\sin(2x)}{6x}$$

$\frac{0}{0}$ a.w.e. (+.5)

$\lim_{x\to 0} \frac{\sin 2x}{3\cdot 2x} = \lim_{x\to 0} \frac{1}{3}\frac{\sin 2x}{2x}$ alg (+.5)

$= \lim_{x\to 0} \frac{1}{3} \cdot \lim_{x\to 0} \frac{\sin 2x}{2x} = \frac{1}{3}$ got it (+.5)

(+.5)

or

table (+.5)

x	$-.1$	$-.0001$	$\geq .0001$
$\frac{\sin 2x}{6x}$			

two sided (+1)

got it (+.5)

(WebHW4 #4)

$$\lim_{x\to 0} \frac{x}{\cos(x)} = \frac{0}{\cos(0)} = \frac{0}{1} = 0$$

(+.5) we can just plug in zero?

$\cos(0) = 1$ (+.5)

got it (+.5)

function (+.5)

or

table (+.5)

two sided (+1)

x	$-.1$	$-.0001$	$\geq .0001$
$\frac{x}{\cos(x)}$			

got it (+.5)

6. [4] (Quiz3 #3) Find the linearization of $f(x) = \frac{1}{\sqrt{x}}$ that is parallel to the line $y - 3 = \frac{-27}{2}(x+5)$

ie, find the line tangent to f when the slope is $-27/2$ (+.5)

Looking for a line $y = mx + b$ (+.5)

m = slope of line $y - 3 = \frac{-27}{2}(x+5)$ (+.5)

$= -27/2$

need to find the point (x, y) so that

(+.5) slope of line tang. to f @ x = slope of line $y - 3 = -\frac{27}{2}(x+5)$

$f'(x) = -27/2$

(+.5) $-\frac{1}{2}x^{-3/2} = -27/2$

$\to x^{-3/2} = 27$ (+.5)

$x = 27^{-2/3} = 1/9$

so passes thru $(1/9, f(1/9))$

or $(1/9, 3)$ (+.5) so

$y - 3 = -\frac{27}{2}(x - 1/9)$ or

got it (+.5)

7. [3] (WebHW10 #5) The radius of a sphere is increasing at a rate of 4mm/s. How fast is the volume increasing when the radius is 30mm?

(+.5) $V = \frac{4}{3}\pi r^3$ where r is the radius and V is the volume

(+1) want $\left.\frac{dV}{dt}\right|_{r=30}$

note: sold the volume formula for +.5

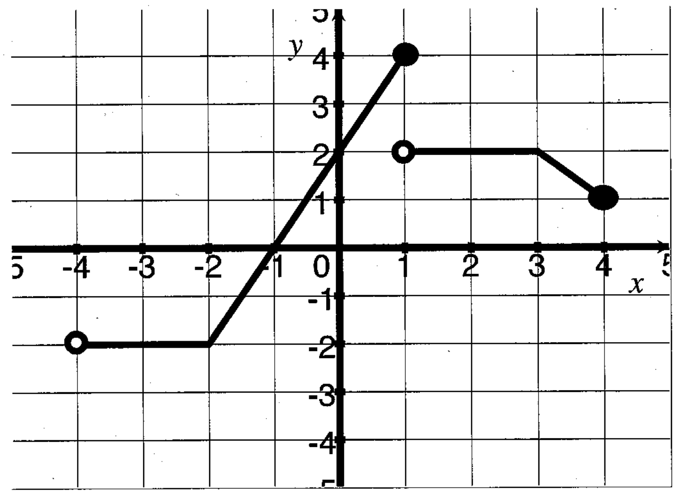
$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$ (+1)

So $\left.\frac{dV}{dt}\right|_{r=30} = \frac{4}{3}\cdot\pi\cdot 3(30)^2 \cdot 4$ (+.5)

$= 14400\pi \frac{mm^3}{s}$

8. Let f be a function where $f(0) = 3$, $f(2) = 5$, $f'(0) = -1$ and g be a piece-wise defined function graphed below.



(a) [1] $g(1)$ 4

(b) [2] (ProductWks #1) $(f \cdot g)'(0)$

Product rule (+.5)

$f(0)g'(0) + f'(0)g(0)$

$3 \cdot 2 + -1 \cdot 2$ (+.5) (+.5)

$6 - 2 = 4$

notation (+.5)

(c) [3] (§3.2 #44) $\frac{d}{dx}\left(\frac{f(x)}{1+g(x)}\right)|_{x=0}$

quotient rule (+.5)
correct quotient (+.5)
notation (+.5)

$\frac{(1+g(x))\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(1+g(x))}{(1+g(x))^2}$

$x=0$

$\frac{(1+g(0))f'(0) - f(0)g'(0)}{(1+g(0))^2} = \frac{3 \cdot (-1) - 3(2)}{(1+2)^2} = \frac{-3-6}{9} = \frac{-9}{9}$ (+.5) (+.5) (+.5)

(d) [3] (§3.4 #65) $\frac{d}{dx}f(g(x))|_{x=0}$

Chain rule (+.5)
correct chain (+.5)

$f'(g(0))g'(0)$ (+.5) (+.5)

$f'(2) \cdot 2$ (+.5)

$5 \cdot 2$

10 got it (+.5)

9. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) Consider a ladder 25ft long leaning against a vertical wall. If the bottom of the ladder slides away front the wall at a rate of 1.25ft/s, answer the following questions.

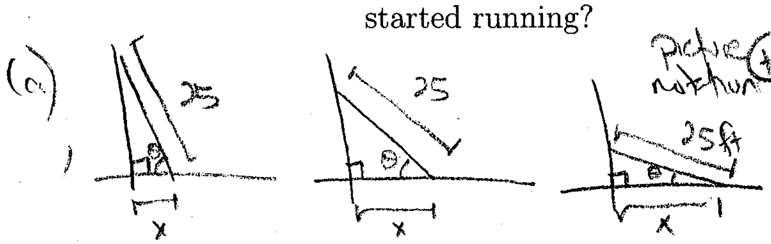
i. [3] Find a formula for how fast the angle between the ladder and the ground is changing t seconds after the bottom of the ladder begins sliding.

ii. [2] How fast is the angle between the ladder and the ground changing when the bottom of the ladder is 72inches from the wall?

(b) Ryan and Stella were being chased by a pack of zombies. At point P they decided to split up. Ryan ran east at about 14 ft/s. Stella waited for 10 seconds to try to draw the zombies towards her and then started to run south at 16ft/s.

i. [3] Find a formula for how fast the distance between them is increasing as a function of t seconds after Stella started running.

ii. [2] How fast is the distance between them increasing one minute after Stella started running?

(a) 

(i) WANT $d\theta/dt$ (+.5)

Sohcahtoa $\cos\theta = \frac{x}{25}$ (+.5)

d/dt: $-(\sin\theta)\frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$ (+1) (+.5)

$\Rightarrow \frac{d\theta}{dt} = \frac{\frac{1}{25}(1.25)}{-\sin\theta}$

(ii) WANT $\left.\frac{d\theta}{dt}\right|_{x=72in} = \left.\frac{d\theta}{dt}\right|_{x=6ft}$ units (+.5)

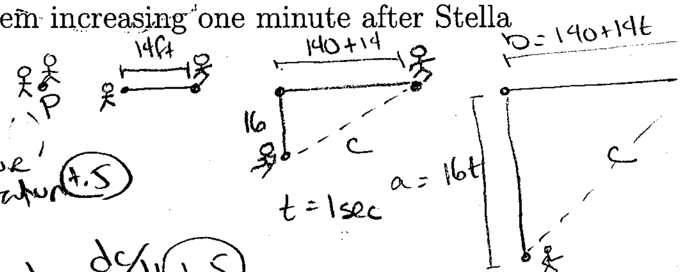
opp, 25, θ, 6ft

$\sin\theta = \frac{opp}{hyp}$ (+1)

$opp^2 + 6^2 = 25^2 \Rightarrow opp = \sqrt{589}$

so $\sin\theta = \frac{\sqrt{589}}{25}$ (+.5)

$\left.\frac{d\theta}{dt}\right| = \frac{\frac{1}{25}(1.25)}{\sqrt{589}} \approx .0515\ \frac{rad}{sec}$

(b) 

(i) want dc/dt (+.5)

$a^2 + b^2 = c^2$ (+.5)

d/dt: $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$ (+1) (+.5)

$\Rightarrow \frac{dc}{dt} = \frac{a\frac{da}{dt} + b\frac{db}{dt}}{c} = \frac{a16 + b\cdot 14}{c}$

(ii) Want $\left.\frac{dc}{dt}\right|_{t=60sec}$ units (+.5)

$140 + 14\cdot 60 = 140 + 840 = 980$

$16\cdot 60 = 960$

$\sqrt{960^2 + 980^2}$ (+1)

$96 \cdot 60 = 960$ $\quad$ $14 \cdot 60 = 840$

$\frac{dc}{dt} = \frac{960\cdot 16 + 980(14)}{\sqrt{960^2 + 980^2}} \approx$ (+.5)

6
10
~~11~~ 21
~~9~~ 20
5 41