Exam 1 **TMath 124** UH Winter 2016

Name:

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

   T F $\frac{4x+3y}{4z} = \frac{x+3y}{z}$

   T F $\lim_{x\to a} f(x) = f(a)$

   T F If f is continuous, then $f'(r)$ exists.

   T F $f'(2)$ is the slope of the line tangent to f at $x = 2$.

   T F If f is continuous, $f(1) = 5$, and $f(4) = -4$, then f has a root between $x = 0$ and $x = 4$

   T F If $\lim_{x\to a} g(x) = 0$, then $\dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ does not exist.

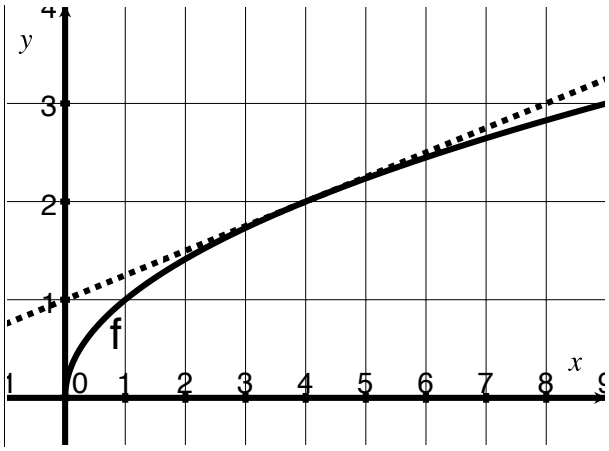
   T F $\lim_{x\to -1}(x^3 + 5x) = -6$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [2] (§2.7 #20) Let f be the function graphed with the solid line and note that the dotted line is the line tangent to f at $x = 4$. Find:

   (a) $f(4)$

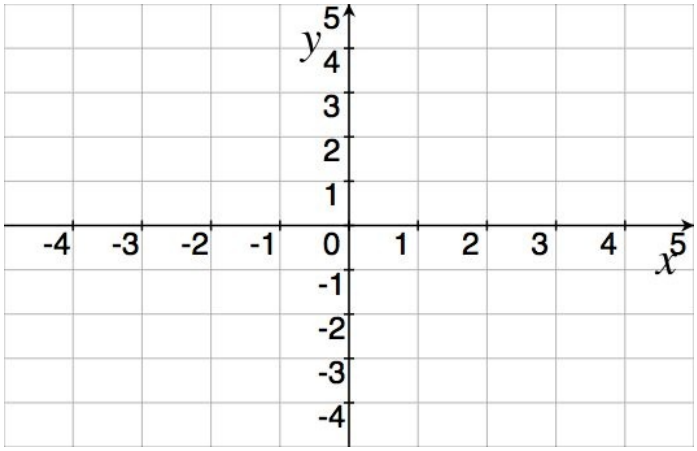
   (b) $f'(4)$



3. Let f be a piece-wise defined function defined by $f(x) = \begin{cases} (x+2)^2 & \text{if } x \leq 0, \\ 2\log_4(x) & \text{if } 0 < x, \end{cases}$

(a) [2] (Quiz1 #1) Graph f on the axes provided.

(b) [1] (§2.2 #12) Determine the values of c for which $\lim_{x \to c} f(x)$ exists.



(c) [3] (WebHW3 #11) Evaluate the following (if they exist!)

$\lim_{x \to 4^+} f(x)$ $\qquad$ $f(0)$ $\qquad$ $\lim_{x \to 0^-} f(x)$

4. [4] Find the limit if it exists, or explain why it does not exist.

(InfLimitsWks #1)

$$\lim_{x \to \infty} \frac{x^2 - 2}{x^3 - 1}$$

(PracticeExam #4)

$$\lim_{x \to \infty} \arctan(x + 2)$$

5. [4] Find the limit if it exists, or explain why it does not exist.

(§2.5 #36)

$$\lim_{x\to\pi} \cos(x + \sin(x))$$

(§2.3 Lecture)

$$\lim_{x\to 0} x^2 \sin\frac{\pi}{x}$$

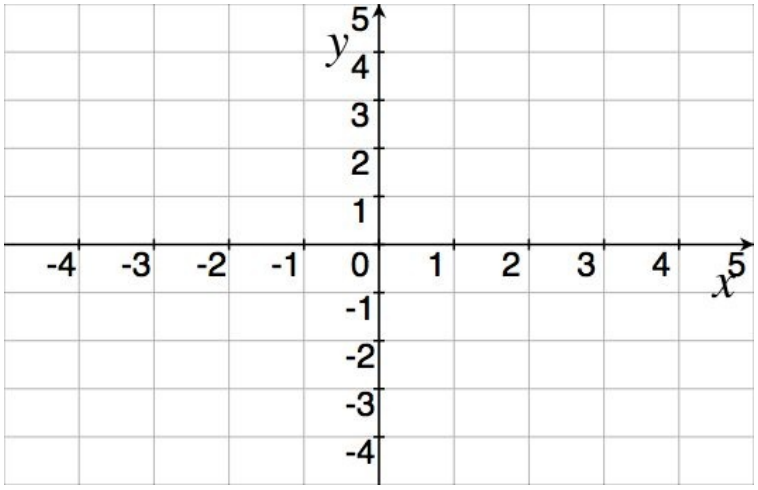
6. [5] (ContWks #6) Sketch a graph of a function α that satisfies *all* of the following:

(a) $\alpha(2) = 2$

(b) $\lim_{x\to 2} \alpha(x) = -3$

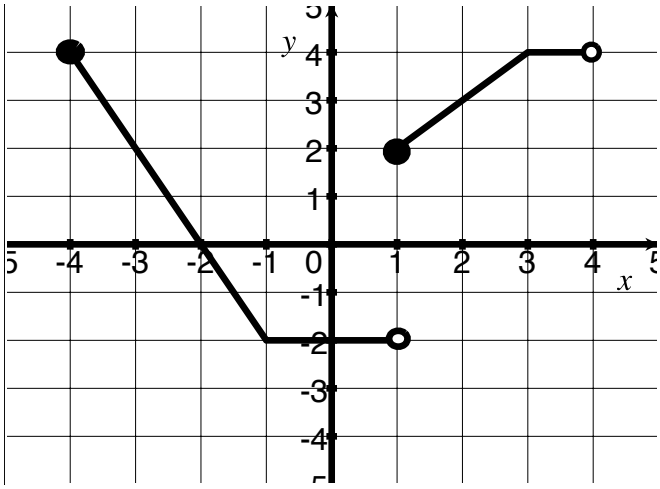
(c) $\lim_{x\to\infty} \alpha(x) = -3$

(d) α is continuous for $-4 \le x \le 1$



7. [3] Write the algebraic rule or the function α you created in the problem above.

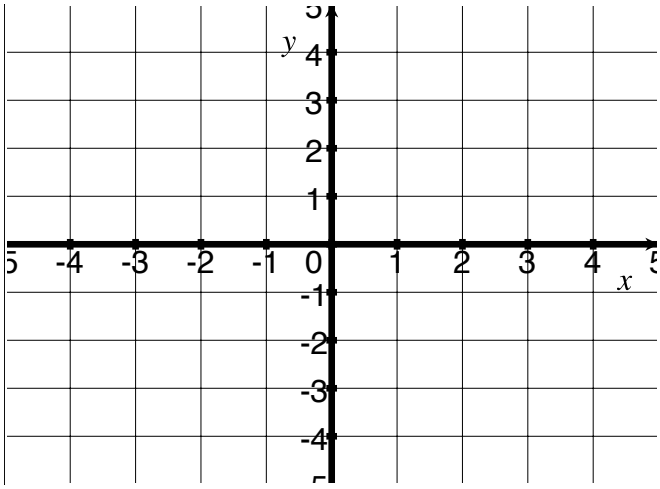
8. Consider the graph of the piece-wise defined function g to answer the following questions



(a) [1] (WebHW2 #1)
$g(1)$

(b) [1] (WebHW2 #1)
$\lim_{x \to 3} g(x)$

(c) [1] (Quiz2 #3)
$g'(3)$



(d) [2] (Quiz2 #3)
$\frac{d}{dx} g|_{x=0}$

(e) [4] (WebHW5 #6)
Draw a graph of $g'(x)$

9. (WebHW5 #3) [5] Let $f(x) = 4x - x^2$. Find the equation for the line tangent to the graph of f, when $x = 1$.

10. Recall Newton's Law of Cooling: If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the objects at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of object.
Find $\lim_{t\to\infty} T(t)$ and interpret the result as a scientist.

(a) [3] Dr. Card's body is found in Joy 109 by a student. At 7:52am the police arrive noting the temperature in the room is 67° F and the bodies temperature is 80°. At 8:02 the police noticed that the body was now 78°F. Let t be the time since the body was found and create a function C to describe Dr. Card's body temperature as a function of t.

(b) [2] Dr. Vanderpool arrives on the scene and insists on having you compute $\lim_{x\to\infty} C(t)$ and explain its meaning to the police officers.