Exam 1 # TMath 124 UH Winter 2016

Name: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

T **F** $\frac{4x+3y}{4z} = \frac{x+3y}{z}$ $\quad \frac{4x+3y}{4z} \neq \frac{x+3y}{z} = \frac{4(x+3y)}{4z} = \frac{4x+12y}{4z}$

T **F** $\lim_{x\to a} f(x) = f(a)$ only if f is continuous

T **F** If f is continuous, then $f'(r)$ exists. f a cont but $f'(a)$ does not exist

T F $f'(2)$ is the slope of the line tangent to f at $x = 2$.

T F If f is continuous, $f(1) = 5$, and $f(4) = -4$, then f has a root between $x = 0$ and $x = 4$ IVT

T **F** If $\lim_{x\to a} g(x) = 0$, then $\dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ does not exist.

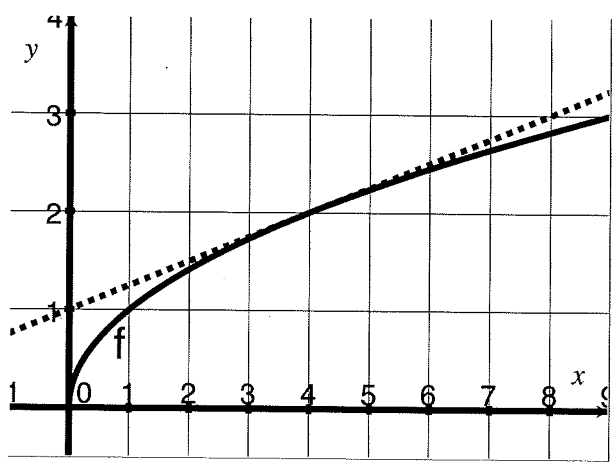
let $g(x) = x-1$ then $\lim_{x\to 1} g(x) = 0$ but $\lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{x-1}} = 3$

T F $\lim_{x\to -1}(x^3 + 5x) = -6$

$(-1)^3 + 5(-1)$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

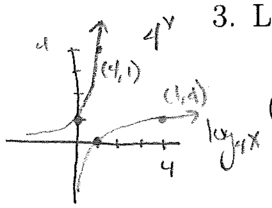
2. [2] (§2.7 #20) Let f be the function graphed with the solid line and note that the dotted line is the line tangent to f at $x = 4$. Find:



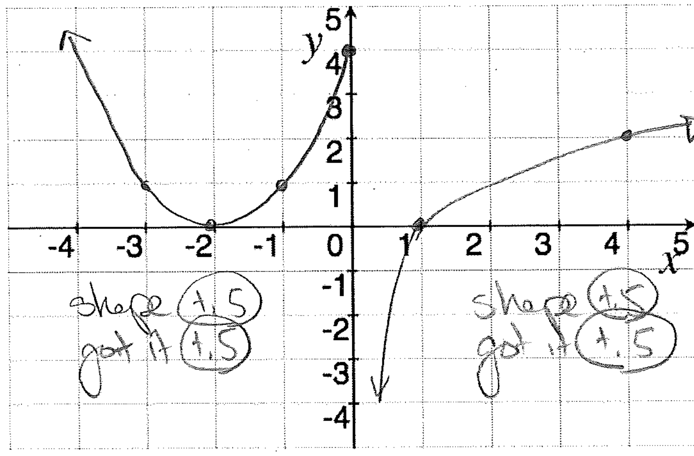
(a) $f(4)$ = y-value if $x=4$ = 2

(b) $f'(4)$ = slope of line tang. to f @ $x=4$ = slope of dotted line = 1/4

3. Let f be a piece-wise defined function defined by $f(x) = \begin{cases} (x+2)^2 & \text{if } x \le 0, \\ 2\log_4(x) & \text{if } 0 < x, \end{cases}$



(a) [2] (Quiz1 #1) Graph f on the axes provided.



(b) [1] (§2.2 #12) Determine the values of c for which $\lim_{x\to c} f(x)$ exists.

all $x \neq 0$

or

$(-\infty, 0) \cup (0, \infty)$

(c) [3] (WebHW3 #11) Evaluate the following (if they exist!)

$\lim_{x\to 4^+} f(x)$	$f(0)$	$\lim_{x\to 0^-} f(x)$
2	4	4

note +.5 if $-\infty$

4. [4] Find the limit if it exists, or explain why it does not exist.

(InfLimitsWks #1)

$$\lim_{x\to\infty} \frac{x^2-2}{x^3-1}$$

notation +.5

alg: $\lim_{x\to\infty} \frac{x^2-2}{x^3-1} \frac{(1/x^3)}{(1/x^3)} = \lim_{x\to\infty} \frac{\frac{x^2}{x^3} - \frac{2}{x^3}}{x^3/x^3 - 1/x^3}$

factor +.5 alg +.5

$= \lim_{x\to\infty} \frac{1/x - 2/x^3}{1 - 1/x^3} = \frac{0}{1} = 0$ +.5

or

$\lim_{x\to\infty} \frac{x^2-2}{x^3-1} = \lim_{x\to\infty} \frac{x^2}{x^3}$ leading term +1 Rachel's Method

$= \lim_{x\to\infty} \frac{1}{x} = 0$ +.5

or

numerically: table +.5 x-values +.5 got it +.5

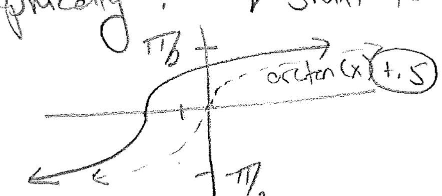
X	100	1,000	10,000
$\frac{x^2-2}{x^3-1}$			

(PracticeExam #4)

$$\lim_{x\to\infty} \arctan(x+2)$$

notation +.5

graphically: shift to the left 2 units +.5



as $x \to \infty$ $y \to \pi/2$ +.5

or

numerically: table +.5

X	100	1000	10,000
arctan(x)			

x-values +.5 got it +.5

5. [4] Find the limit if it exists, or explain why it does not exist.

(§2.5 #36)

$\lim_{x\to\pi} \cos(x + \sin(x))$

b/c sine & cosine are continuous and the composition of cont functions are continuous

$= \cos(\pi + \sin(\pi))$ plug in (.5)

$= \cos(\pi + 0)$ eval trig (+1)

$= \cos \pi = -1$ notation (.5)

(§2.3 Lecture)

$\lim_{x\to 0} x^2 \sin \frac{\pi}{x}$ sense (.5)

Algebraically! Notice $-1 \le \sin \pi/x \le 1$ (.5)

$\Rightarrow -x^2 \le \sin \pi/x \le x^2$

Since $\lim_{x\to0} -x^2 = 0 = \lim_{x\to0} x^2$ (+.5)

by the squeeze thm $\lim_{x\to0} x^2 \sin \pi/x = 0$ (+.5)

or

table (+.5) notation (+.5) x: $-.001$ | $.001$; y

2-sides (+1)

6. [5] (ContWks #6) Sketch a graph of a function α that satisfies *all* of the following:

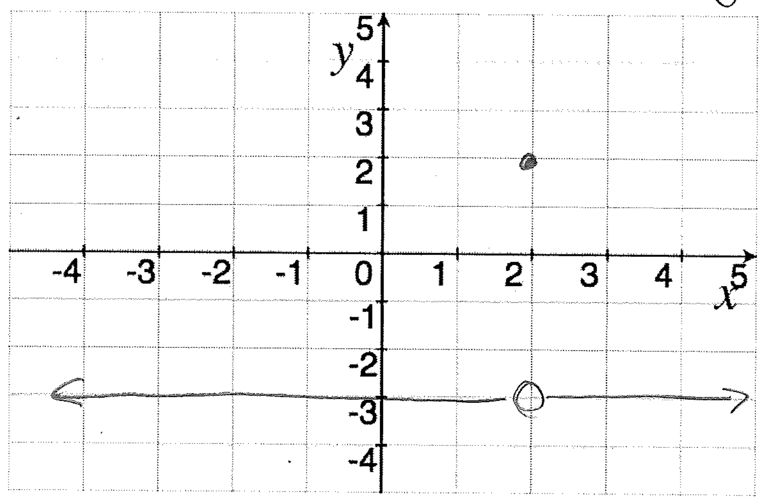
(a) $\alpha(2) = 2$

(b) $\lim_{x\to 2} \alpha(x) = -3$

(c) $\lim_{x\to\infty} \alpha(x) = -3$

(d) α is continuous for $-4 \le x \le 1$

Note: There are MANY answers

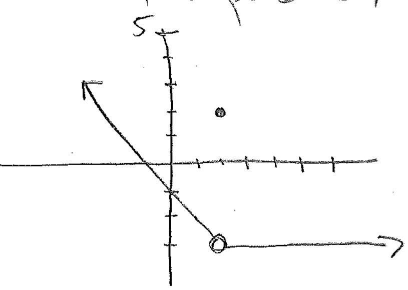


7. [3] Write the algebraic rule or the function α you created in the problem above.

$$\alpha(x) = \begin{cases} 2 & \text{if } x = 2 \\ -3 & \text{if } x \neq 2 \end{cases}$$

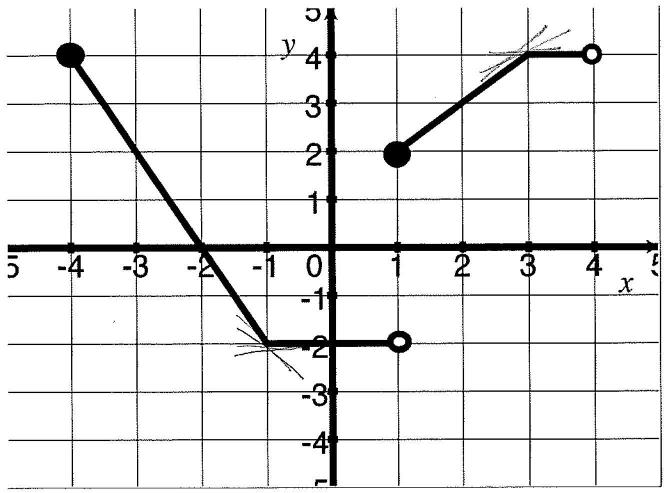
Sketch (+.5) function (+.5) match (+2)

other possible answers



$$\alpha(x) = \begin{cases} -x-1 & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ -3 & \text{if } 2 < x \end{cases}$$

8. Consider the graph of the piece-wise defined function g to answer the following questions



(a) [1] (WebHW2 #1)
$g(1)$

2

(b) [1] (WebHW2 #1)
$\lim_{x\to 3} g(x)$

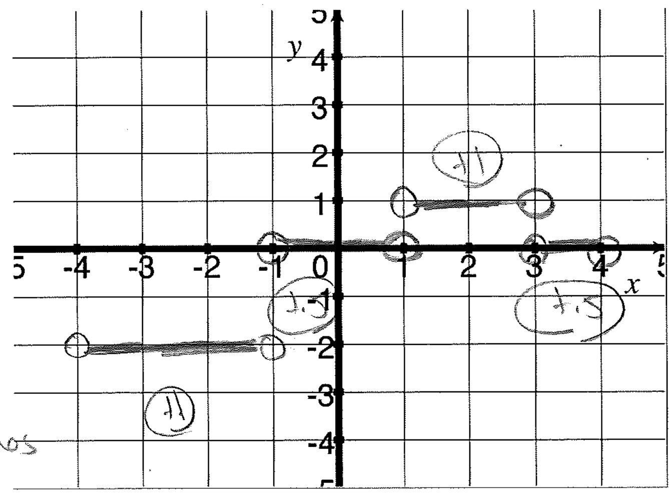
4

(c) [1] (Quiz2 #3)
$g'(3)$ DNE

(d) [2] (Quiz2 #3)
$\frac{d}{dx} g|_{x=0}$

0 -2 $+.5$

(e) [4] (WebHW5 #6)
Draw a graph of $g'(x)$



endpoints (+.5)

slope of tang lines (+.5)

9. (WebHW5 #3) [5] Let $f(x) = 4x - x^2$. Find the equation for the line tangent to the graph of f, when $x = 1$.

equation of a line $y = mx+b$ or $y - y_1 = m(x - x_1)$ (+.5)

m = slope of line tang. to f @ $x=1$

$= f'(1)$ (+.5)

$= \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{[4(1+h)-(1+h)^2]-[4(1)-1^2]}{h}$ (+1)

$= \lim_{h\to 0} \frac{\cancel{4}+4h-\cancel{1}-2h-h^2-\cancel{4}+\cancel{1}}{h} = \lim_{h\to 0} \frac{\cancel{h}(4-2-h)}{\cancel{h}}$

$= \lim_{h\to 0} 2-h = 2$ alg (+1) limit eval (+.5)

Passes thru the point $(1, f(1))$ or $(1,3)$ (+.5)

So (+.5) $3 = 2(1)+b \Rightarrow b = 1$

Eq of line: $y = 2x+1$ (+.5)

or

equation of a line $y = mx+b$ (+.5)

m = slope of line tang to f @ $x=1$

$= f'(1)$ (+.5)

notice $f'(x) = 4(1)x^0 - 2x$
$= 4 - 2x$ (+.5)

So $f'(1) = 4 - 2(1) = 2$ (+1)

line passes thru $(1,3)$ (+.5)

So $3 = 2(1)+b \Rightarrow b = 1$ (+.5)

Eq of line: $y = 2x+1$

10. Recall Newton's Law of Cooling: If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the objects at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of object.
Find $\lim_{t\to\infty} T(t)$ and interpret the result as a scientist.

(a) [3] Dr. Card's body is found in Joy 109 by a student. At 7:52am the police arrive noting the temperature in the room is 67° F and the bodies temperature is 80°. At 8:02 the police noticed that the body was now 78°F. Let t be the time since the body was found and create a function C to describe Dr. Card's body temperature as a function of t.

(b) [2] Dr. Vanderpool arrives on the scene and insists on having you compute $\lim_{x\to\infty} C(t)$ and explain its meaning to the police officers.

(a) $T_s = 67°F$ (+.5)

$D_0 = 80° - 67° = 13°F$ (+.5)

t time since 7:52

$\Rightarrow$ $C(t) = 67 + 13e^{-kt}$ (+.5)

we need to figure out k when $t=10$, $C(10)=78$ so

when $t = 10$ $C(10) = 78°$

notation (+.5)

$78 = \cancel{67} + 13e^{-k10}$ & solve for t

$-67 \quad -\cancel{67}$

alg (+1)

$\frac{11}{13} = \frac{\cancel{13}e^{-k10}}{\cancel{13}}$

$11/13 = e^{-k10}$

$\frac{\ln(11/13)}{-10} = \frac{-k10}{-10}$

$k = \frac{-1}{10}\ln(11/13) \approx .016705$

So function: $67 + 13e^{-.0167t}$

(b) $\lim_{x\to\infty} C(t) = \lim_{x\to\infty}(67 + 13e^{-.0167t}) = \lim_{x\to\infty} 67 + \underbrace{\lim_{x\to\infty} \frac{13}{e^{.0167t}}}_{\text{equal to zero/alg}} = 67$ (+.5)

(+.5)

limit laws (+1)

note: Even if you didn't have (a) $\lim_{x\to\infty} T_s + D_0e^{-kt} = \lim_{x\to\infty} T_s + \lim_{x\to\infty} \frac{D_0}{e^{kt}} = T_s$