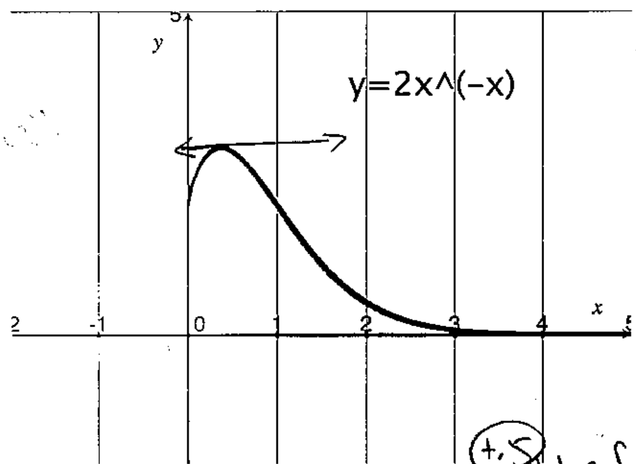
Key

TMATH 124: Quiz 4

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. No calculators or notes are allowed.

1. Consider the function $f(x) = 2x^{-x}$, graphed below.



(a) [4] (WebHW12 #10) Find $f'(x)$

$y = 2x^{-x}$

intro ln (+.5)

properties (+1) $\ln y = \ln 2x^{-x}$

$\ln y = \ln 2 + \ln x^{-x}$

$\ln y = \ln 2 - x\ln x$

$\frac{d}{dx}$ (+.5) (+.5) (+1)

$\frac{1}{y}\frac{dy}{dx} = 0 - (x(\ln x)' + (x)'\ln x)$

$\frac{1}{y}\frac{dy}{dx} = -x\left(\frac{1}{x}\right) - (1)\ln x$

(+.5) solve for $\frac{dy}{dx}$

$\frac{dy}{dx} = y\left[-1 - \ln x\right]$

or $2x^{-x}(-1 - \ln x) = \frac{2}{x^x}(-1-\ln$

(b) [2] (§4.1 #66) Use calculus to find the exact maximum value between 0 and 5.

want to find where $f'(x) = 0$ AND $f'(x)$ DNE

$f'(x) = 0$ (+.5)

$2x^{-x}(-1-\ln x) = 0$ alg (+1)

$\Rightarrow 2x^{-x} = 0$ or $-1 - \ln x = 0$

$\Rightarrow$ no solution $\quad \Rightarrow -\ln x = 1$

$\Rightarrow \ln x = -1$

$\Rightarrow e^{-1} = x$

or $x = 1/e$

$f'(x)$ DNE

when den $= 0$ or $\ln$ is not defined

$x^x = 0$ $\quad \Rightarrow x \leq 0$

$\Rightarrow$ no solution

notation (+.5)

So when $x = 1/e \approx .37$

value of $f\left(\frac{1}{e}\right) = 2\left(\frac{1}{e}\right)^{-\frac{1}{e}} = 2e^{1/e} \approx 2.89$

(c) [1] Find the equation of the line tangent to f that has a slope of zero.

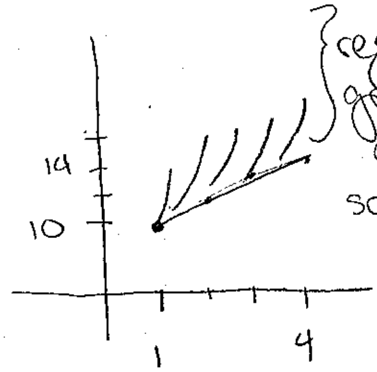
$y = 2e^{1/e}$

eq of line (+.5)

consistent w/ (b) (+.5)

2. [3] (§4.2 #23) If $g(1) = 10$ and $g'(x) \geq 2$ for all x between 1 and 4, how small can $g(4)$ possible be? Justify your answer.

16 (+1)
reasoning (+1)
sense/clarity (+1)



region that the graph of g can exist in.
so $g(4) \geq 16$

or

Since the rate of g' is greater than 2 units/time the growth over 3 time units (4-1) must be greater than $3 \cdot 2$ or 6 units.

Thus if $g(1) = 10$, by $x = 4$, $g(4)$ must have grown more than 6 units

$\Rightarrow g(4) \geq 10 + 6 = 16$

or

Recall the mean value theorem applied to this function g on the interval $[1, 4]$. There exists a number c between 1 & 4 so that

$$g'(c) = \frac{g(4) - g(1)}{4-1} \quad \text{or} \quad g'(c) = \frac{g(4) - 10}{3}$$

Since $2 \leq g'(x) \leq ?$ for all #'s between 1 & 4

$2 \leq g'(c) \leq$ so the above implies

$$2 \leq \frac{g(4) - 10}{3} \quad \text{or} \quad 6 \leq g(4) - 10 \quad \text{or} \quad 16 \leq g(4)$$