Key

EXAM 1

TMath 124

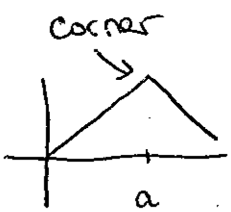
Winter 2015

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function and x and y be positive numbers.

T **F** $\frac{9}{\sqrt{x}} = 9x^{\frac{1}{2}}$. $9x^{\frac{1}{2}} = 9\sqrt{x}$ $\frac{9}{\sqrt{x}} = 9x^{-1/2}$

T **F** If $\lim_{h\to 0} g(h) = 0$, then $\lim_{h\to 0} \frac{f(h)}{g(h)}$ does not exist. $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$ exists (is $2x$)

T F If f is continuous and $\lim_{x\to -2} f(x) = 5.2$, then $f(-2) = 5.2$.



T **F** If $\lim_{x\to a} f(x) = f(a)$, then $f'(a)$ exists.

T F The parabola is the graph of a differentiable function.

T **F** $\frac{d}{dx}(e^x) = xe^{x-1}$ $\frac{d}{dx}(e^x) = e^x$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [5] (Quiz 2 #1) Sketch the graph of an example function f that satisfies the following: (+1)

(a) f is *not* continuous at $x = 0$ (+1)

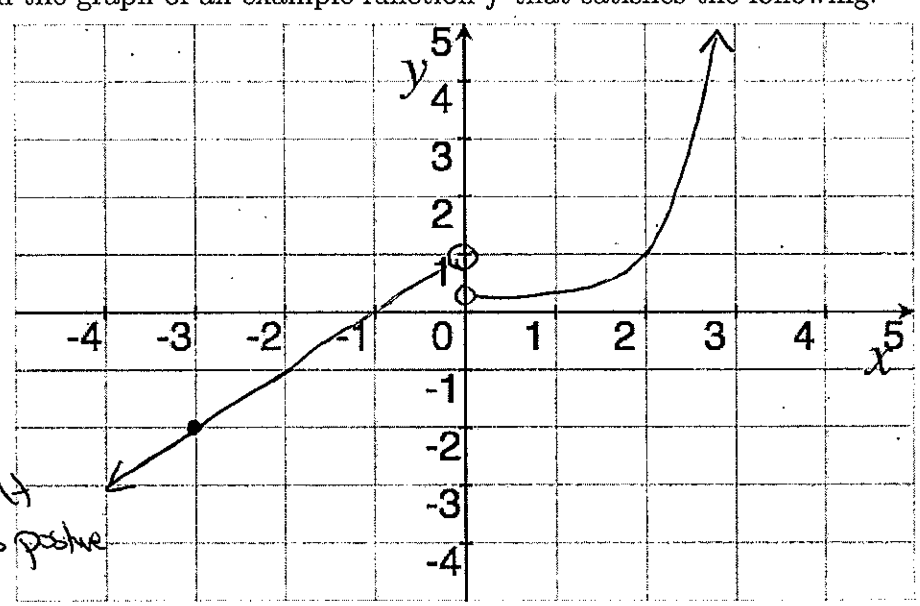
(b) $\lim_{x\to 3^-} f(x) = \infty$ (+1)

(c) $f(-3) = -2$ (+1)

(d) $f'(-3) > 0$ (+1)

Slope of line tangent to f at x=-3 is positive

note: there are many answers



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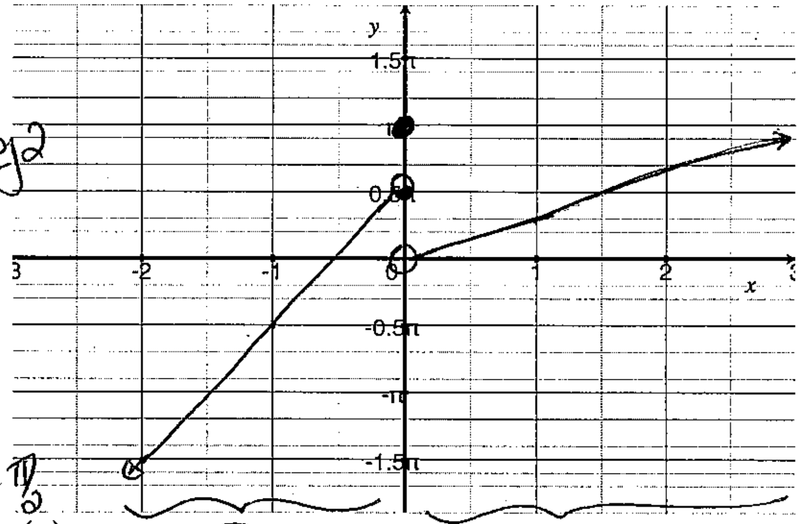
3. Let

$$g(x) = \begin{cases} \pi x + \frac{\pi}{2} & \text{if } x < 0 \\ \pi & \text{if } x = 0 \\ 2\arctan(x) & \text{if } 0 < x \end{cases}$$

↳ vertical stretch by 2



(+1) (+1)

(a) [3] (Quiz1 #1) *Carefully* graph g below.

end points (+1)

(b) [1] (§2.2 #5) Estimate $\lim_{x\to 0^-} g(x)$

$\frac{\pi}{2}$ or $\pi(0)+\frac{\pi}{2}$

(c) [2] (§2.6 #4) Estimate $\lim_{x\to\infty} g(x)$

(+1) π inf limit +1

(d) [2] (WebHW5 #6) Estimate $g'(-1)$ (+1) slope of tangent line when $x=-1$

about π (+1)

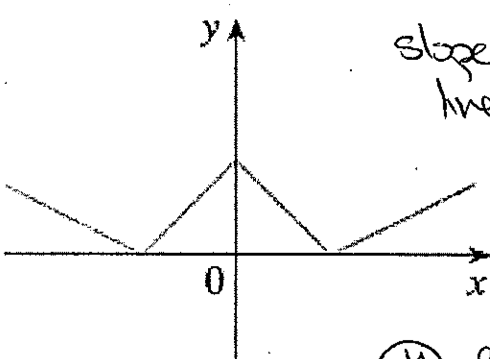
(e) [2] (§2.3 #2) Estimate $\lim_{x\to -2}\left[\frac{2}{3}g(x)+2\right]$ $= \frac{2}{3}\lim_{x\to -2} g(x) + 2$ (+1)

$= \frac{2}{3}\cdot -\frac{3}{2}\pi + 2 = -\pi + 2 \approx -1.14$ (+1)

4. [4] (WebHW6 #4) Consider the function m graphed on the left. Sketch m'.

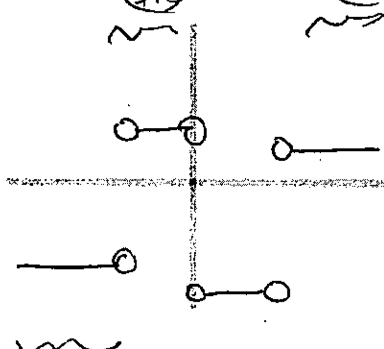
notation (+.5)

slope of tangent lines... (+.5)



(+1) endpoints

(+.5) (+.5)



(+.5) (+.5)

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5. [12] (§2.3 #2, WebW4 #10, Day 2 extra practice, & PracticeExam #4) Find the limit or explain why it does not exist.

Notation (+1) +.5 each

$$\lim_{x\to -1}(x^3-2x+3)$$

$= \lim_{x\to -1} x^3 - 2\lim_{x\to -1} x + 3$ (+1)

$= (-1)^3 - 2(-1) + 3$

$= -1 + 2 + 3 = 4$ (+1)

or

polynomials are continuous so $(-1)^3 - 2(-1) + 3 = 4$

approach (+1) got it (+1)

$$\lim_{x\to\infty}\frac{7x-2}{1+2x}$$

ratio of leading terms $\frac{7}{2}$

or

$\lim_{x\to\infty} \dfrac{7x-2}{2x+1} \cdot \dfrac{1/x}{1/x}$

$= \lim_{x\to\infty} \dfrac{7x/x - 2/x}{2x/x + 1/x} = \lim_{x\to\infty} \dfrac{7 - \cancel{2/x}}{2 + \cancel{1/x}}$

$= 7/2$

or

x	100	1,000	10,000
y			

notation (+1) +.5 each

$$\lim_{h\to 0}\frac{(2+h)^{-1}-2^{-1}}{h}$$

neg exp (+1)

$= \lim_{h\to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h}$

fraction sub (+1)

$= \lim_{h\to 0} \dfrac{\left(\frac{2-(2+h)}{2(2+h)}\right)}{h}$

simplify order of operations & arithmetic (+1)

$= \lim_{h\to 0} \dfrac{-h}{2(2+h)} \div \dfrac{h}{1}$

$= \lim_{h\to 0} \dfrac{-\cancel{h}}{2(2+h)} \cdot \dfrac{1}{\cancel{h}}$

eval limit (+1)

$= \lim_{h\to 0} \dfrac{-1}{2(2+h)}$

$= \dfrac{-1}{2(2+0)} = \dfrac{-1}{4}$

approach (+1) got it (+1)

$$\lim_{x\to 0}\frac{1}{x^2}\sin x$$

Recall $-1 \le \sin x \le 1$

b/c $1/x^2$ is positive (or zero)

$\Rightarrow -\frac{1}{x^2}\cdot 1 \le \frac{1}{x^2}\sin x \le \frac{1}{x^2}\cdot 1$

Since $\lim_{x\to 0} \frac{1}{x^2} = \infty$ Avg?

"$\infty \cdot 0$"

x	.1	.01	.005
$\frac{1}{x^2}\cdot\sin x$	9.98	99.99	199.99

x	-.1	-.01	-.005
$\frac{1}{x^2}\sin x$	-9.98	-99.99	-199.99

so ∞ on one way & $-\infty$ the other

DNE

6. (Day 6 extra practice) Let $f(x) = 1 + 2e^x - 3x$.

(a) [5] Find the equation for the line tangent to the graph of f, when $x = 0$.

(+.5) Looking for $y = mx + b$

m = slope of line tangent to f at $x = 0$

(+.5) $= f'(0)$

plug in (+1) $= 2e^0 - 3 = 2 - 3 = -1$

Finding $f'(x)$:

(+1.5) $\frac{d}{dx}[1 + 2e^x - 3x] = \frac{d}{dx}[1] + \frac{d}{dx}[2e^x] - \frac{d}{dx}[3x]$

$= 0 + 2\frac{d}{dx}[e^x] - 3 = 2e^x - 3$

(+1) passes thru the point $(0, f(0)) = (0, 1 + 2e^0 - 3 \cdot 0) = (0, 3)$

so

$y - 3 = (-1)(x - 0)$ or $3 = (-1)(0) + b$

$\Rightarrow 3 = b$

notation (+.5)

so $y = -1x + 3$

(b) [3] At what point of f is the tangent line parallel to $3x - y = 5$?

want to find when

(+.5) slope of line tangent to f at $x = ?$ = slope of $3x - y = 5$

i.e. when does

$f'(?) =$ slope of $3x - 5 = y$

i.e. when does

(+.5) $f'(?) = 3$

from above $f'(x) = 2e^x - 3$ so

(+.5) $2e^? - \cancel{3} = 3$

$+\cancel{3} \quad +3$

$\frac{\cancel{2}e^?}{\cancel{2}} = \frac{6}{2}$

$\to e^? = 3$

$\ln e^? = \ln 3$

$? = \ln 3$

alg solve (+.5)
use ln (+1)

7. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§2.6 #63) Under certain assumptions the velocity $v(t)$ of a falling raindrop at time t is:

$$v(t) = v^*(1 - e^{\frac{-gt}{v^*}})$$

where g is the acceleration due to gravity (9.8 m/s^2), and v^* is a constant.

i. [3] Find $\lim_{t\to\infty} v(t)$.

ii. [2] Interpret the answer given in (i) as a scientist and explain what v^* is in everyday language.

(b) [5] (Story Wks #6) The shuttle Discovery launched the Hubble Space Telescope April 24th 1990.
The shuttle's distance traveled from liftoff ($t = 0$) to jettisoning the rocket boosters ($t = 126$s) was well modeled by the function:

$$0.0003255t^4 - 0.03009667t^3 + 11.805t^2 - 3.083t$$

i. [2] Find a function that describes the velocity of the shuttle.

ii. [1] Find a function that describes the acceleration of the shuttle.

iii. [2] Out line the steps you would do to identify the maximum acceleration obtained by the shuttle in the first 126 seconds. You do not need to complete these steps, just outline them clearly!

start (+.5)

(a) i) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty} v^*(1-e^{-gt/v^*})$ (+.5)

$= v^* \lim_{t\to\infty} \left(1 - \frac{1}{e^{gt/v^*}}\right)$ properties (+1)

$= v^* \left(1 - \lim_{t\to\infty} \frac{1}{e^{gt/v^*}}\right)$

looks like 1/BIG = little (+.5)

$= v^*(1-0) = v^*$ (+.5)

ii) v^* is the speed that the raindrop approaches (+1) also known as terminal velocity (+1)

start (+.5)

(b) i) recall velocity = d/dt(position) (+.5)

$\Rightarrow v(t) = [.0003255t^4 - .03009667t^3 + 11.805t^2 - 3$

$\Rightarrow v(t) = .001302t^3 - .09029001t^2 + 23.61t - 3.0$ (+1)

ii) recall acceleration = d/dt(velocity) (+.5)

$\Rightarrow a(t) = \frac{d}{dt}[.001302t^3 - .09029001t^2 + 23.61t - 3.08$

$= .003906t^2 - .18058002t + 23.61$ (+.5)

right track good

iii) Note acceleration is a parabola (opening up) (+1)

we can graph the parabola (after finding the vertex) & look at the section between 0 & 126 sec

or

B/c we know the parabola is opening up - we know the max will be on the end points. Test ... a(0) & a(126)