Key

EXAM 2 — TMath 124 — Winter 2015

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function and x and y be positive numbers.

T F $(x+y)^2 = x^2 + y^2$ — $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

T F $\sin(5x) = 5\sin(x)$ — $\sin(5x)$ is much more complicated

T F $(\frac{1}{x})' = -x^{-2}$ — $\left(\frac{1}{x}\right)' = (x^{-1})' = -x^{-2}$

T F If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$. Theorem

T F $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$ — Chain rule

T F $\frac{d}{dx}(2^x) = 2^{x-1}$ — $\frac{d}{dx}(2^x) = 2^x \ln 2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

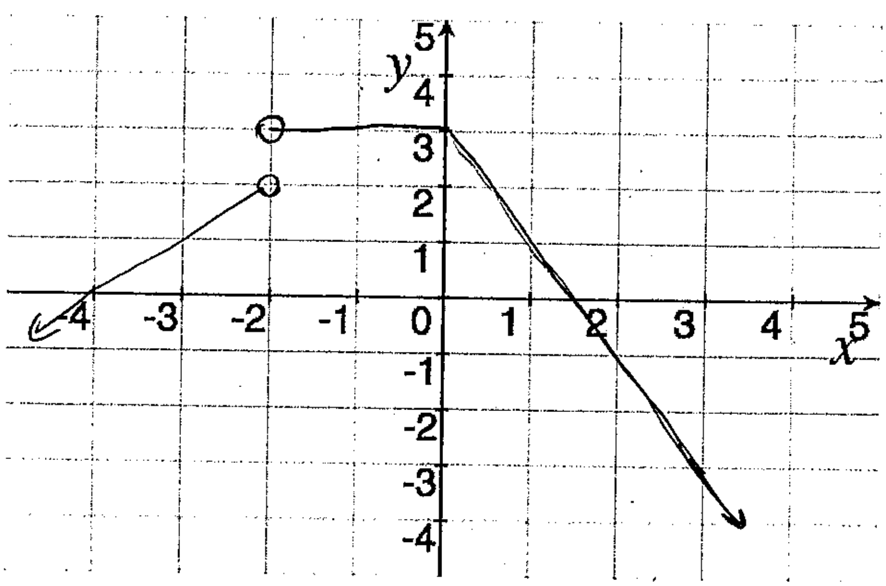
2. [5] (PracticeExam #2) Sketch the graph of an example function f that satisfies: (±1)

(a) f is continuous at $x = 0$ ✓ (x)

(b) $f'(0)$ does not exist ✓ (1)

(c) $\frac{df}{dx}|_{x=2} = -2$ ✓ (x)

(d) $f'(-3) > 0$ (x)



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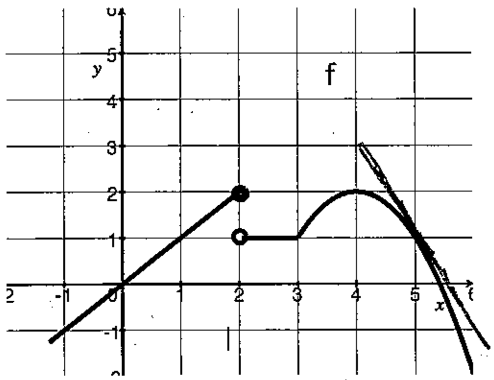
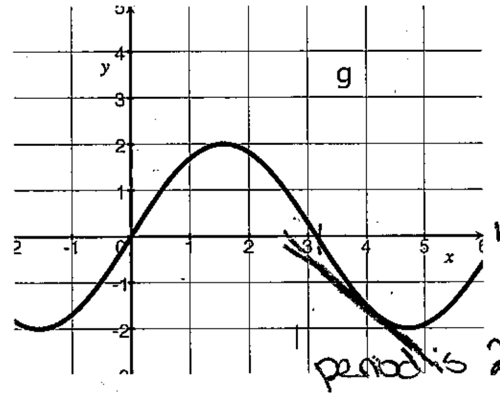
3. The function f is a piece-wise defined function comprised of lines and a parabola that has only been shifted graphed below. The function g is also graphed below and is of a sine curve that has been vertically stretched.





period is 2π

(a) [4] (PracticeExam2 #2) Find the (algebraic) formula for f and g in the form: shift (+.5)

$$f(x) = \begin{cases} x \quad (+.5) & \text{if } -1.25 \le x \le 2 \\ 1 \quad (+.5) & \text{if } 2 < x < 3 \\ -(x-4)^2+2 & \text{if } 3 \le x \end{cases}$$

parabola (+.5) vert shift (+.5) horiz shift (+.5)

$g(x) = 2\sin(x)$ (+.5) (+.5)

(b) Estimate the following (if they exist):

[3] (WebHW8 #8)

$(f \cdot g)'(4)$ product rule (+.5) right (+.5)

$f'(4)g(4) + f(4)g'(4)$

$\approx 0 \cdot -1.5 + 2 \cdot (-1.5)$ (+.5) (+.5) (+.5)

≈ -3 graph reading

(+.5) notation

[2] (CC10)

$\lim_{x\to 0} \frac{g(x)}{f(x)} = \lim_{x\to 0} \frac{2\sin x}{x}$ (+1)

$= 2 \lim_{x\to 0} \frac{\sin x}{x}$ (+.5)

$= 2 \cdot 1 = 2$ (+.5)

if looking for limit (+.5)

[3] (Day9 ExtraWks)

$(f \circ f)'(5)$ chain rule (+.5) right (+.5) notation (+.5)

$f'(f(5))\, f'(5)$

$f'(1) \quad (-2)$

(+.5) (+.5)

$1\,(-2) = -2$ (+.5)

[3] (§3.2 #44d)

$\frac{d}{dx}\left(\frac{g(x)}{1+f(x)}\right)\Big|_{x=0}$ quotient rule (+.5) right (+.5)

$$\frac{(1+f(x))\frac{d}{dx}(g(x)) - g(x)\frac{d}{dx}(1+f(x))}{(1+f(x))^2}$$ (+.5)

$$\frac{(1+f(x))g'(x) - g(x)f'(x)}{(1+f(x))^2}$$

$$\frac{(1+0)(2) - 0(1)}{(1+0)^2} = \frac{2}{1} = 2$$ (+.5) (+.5) (+.5)

4. Find $\frac{dy}{dx}$ of the following:

[3] (Day11 ExtraWks)

$$y = \frac{x^2+5x}{\sqrt{x^3+3}} = \frac{(x^2+5x)}{(x^3+3)^{1/2}}$$

quotient (+.5)
right (+.5)

$$\frac{(x^3+3)^{1/2}\frac{d}{dx}(x^2+5x) - (x^2+5x)\frac{d}{dx}\left((x^3+3)^{1/2}\right)}{\left[(x^3+3)^{1/2}\right]^2}$$

(+.5)

$$= \frac{(x^3+3)^{1/2}\underbrace{(2x+5)}_{+.5} - (x^2+5x)\underbrace{\tfrac{1}{2}(x^3+3)^{-1/2}(3x^2)}_{+1}}{x^3+3}$$

or

product (+.5)
right (+.5)

$$\left[(x^2+5x)(x^3+3)^{-1/2}\right]'$$

$$= (x^2+5x)\frac{d}{dx}\left((x^3+3)^{-1/2}\right) + \frac{d}{dx}(x^2+5x)\,(x^3+3)^{-1/2}$$

form (+.5)

$$= (x^2+5x)\underbrace{\left(-\tfrac{1}{2}\right)(x^3+3)^{-3/2}(3x^2)}_{+1} + \underbrace{(2x+5)}_{+.5}(x^3+3)^{-1/2}$$

[2] (CC10)

$y = 2^x + x^2 + 2^2$

$$y' = (2^x)' + (x^2)' + (2^2)'$$

$$= \underbrace{2^x(\ln 2)}_{+.5} + \underbrace{2x}_{+.5} + \underbrace{0}_{+.5}$$

do each piece (+.5)

[3] (§3.4 #20)

$y = (3x-1)^4(2x+1)^{-3}$

quotient (+.5)
right (+.5)

$$\left[\frac{(3x-1)^4}{(2x+1)^3}\right]'$$

$$\frac{(2x+1)^3\left[(3x-1)^4\right]' - (3x-1)^4\left[(2x+1)^3\right]'}{\left[(2x+1)^3\right]^2}$$

$$\frac{(2x+1)^3\,\underbrace{4(3x-1)^3\cdot 3}_{+1} - (3x-1)^4\,\underbrace{3(2x+1)^2 2}_{+1}}{\left[(2x+1)^3\right]^2}$$

or

product (+.5)
right (+.5)

$$(3x-1)^4\left[(2x+1)^{-3}\right]' + \left[(3x-1)^4\right]'(2x+1)^{-3}$$

$$(3x-1)^4\underbrace{(-3)(2x+1)^{-4}(2)}_{+1} + \underbrace{4(3x-1)^3(3)}_{+1}(2x+1)^{-3}$$

[4] (WebHW10 #3)

$(8\cos(x))(\sin(y)) = 6$

product (+.5)
right (+.5)

$$\underbrace{0}_{+.5} = 8\cos(x)\frac{d}{dx}(\sin y) + \frac{d}{dx}(8\cos(x))\sin y$$

$$0 = 8\cos(x)\underbrace{(\cos y)}_{+.5}\underbrace{\left(\frac{dy}{dx}\right)}_{+.5} + \underbrace{8(-\sin x)}_{+.5}\sin y$$

$$\Rightarrow 0 = \frac{dy}{dx}8\cos x\cos y - 8\sin x\sin y$$

$$\Rightarrow 8\sin x\sin y = \frac{dy}{dx}8\cos x\cos y$$

$$\Rightarrow \frac{dy}{dx} = \frac{8\sin x\sin y}{8\cos x\cos y}$$

or $\tan x\tan y$

solve for $\frac{dy}{dx}$ (+1)

5. [3] (Quiz 3) Find where the function $f(x) = \frac{5x+2}{3x^2}$ has a horizontal tangent line.

find x values so that

(+.5) slope of line tangent to f = slope of horizontal line

$f'(x) = 0$

$\frac{15x^2 - 30x^2 - 12x}{9x^4} = 0$

(+1.5) finding f':

$$\frac{3x^2(5x+2)' - (5x+2)(3x^2)'}{(3x^2)^2}$$

$$\frac{3x^2(5) - (5x+2)6x}{9x^4}$$

algebra (1)

$\Rightarrow \frac{-15x^2 - 12x}{9x^4} = 0 \Rightarrow -15x^2 - 12x = 0 \Rightarrow x(-15x-12) = 0$

$\cancel{x = 0}$ not in domain of f? or $-15x - 12 = 0 \Rightarrow x = \frac{-12}{15} = \frac{-4}{5}$

6. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

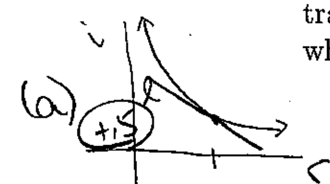
(a) (WordProblemWks #10) If a current i passes through a resistor with resistance r, Ohm's Law states that the voltage drop is $v = ri$. Assume that voltage remains a constant 20 volts. An unreliable resistor claims a resistance of 10 ohms but may be off by up to 1.5 ohms. Use the linear approximation to approximate the error when calculating i. $i = v/r = 20/r$

(b) (§3.5 #26) A certain race track takes the shape of an ellipse defined by the equation

$$x^2 + 2xy + 2y^2 = 10$$

During a race, one of the cars tires come off when it is at the position $(2, 1)$ on the above curve, and the car slides in a direction along the tangent line to the track at that point. Find the equation of the line describing the path of the car when its tires come off.

(a) (+.5)



(+.5) find the eq of ℓ to estimate error

(+.5) m = slope of line tangent to $i = 20/r$ at $r = 10$
$= i'(10)$

(+.5) finding $i'(r) = (20/r)' = (20r^{-1})' = -20r^{-2}$

(+.5) so $m = -20(10)^{-2} = \frac{-20}{100} = \frac{-1}{5}$

(+.5) passes thru $(10, \frac{20}{10}) = (10, 2)$

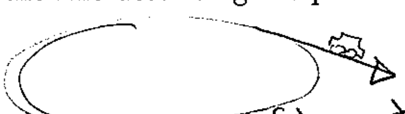
So line is $i - 2 = \frac{-1}{5}(r - 10)$

or $i = \frac{-1}{5}r + 4$

(+1) A reasonable estimate of the range is

$(\frac{-1}{5})(11.5) + 4$ to $(\frac{-1}{5})(8.5) + 4$

(b)



find the equation of the tangent line?

(+.5) looking for $y = mx + b$

(+.5) m = slope of line tangent to curve at $x = 2$
$= dy/dx|_{x=2}$

finding dy/dx:

$2x + \underbrace{2x\frac{dy}{dx} + 2y}_{(+1)} + \underbrace{4y\frac{dy}{dx}}_{(+.5)} = 0$

$\Rightarrow \frac{dy}{dx}2x + \frac{dy}{dx}4y = -2x - 2y$

$\Rightarrow \frac{dy}{dx} = \frac{-2x - 2y}{2x + 4y}$

(+.5) $m = \frac{-2(2) - 2(1)}{2(2) + 4(1)} = \frac{-6}{8} = \frac{-3}{4}$

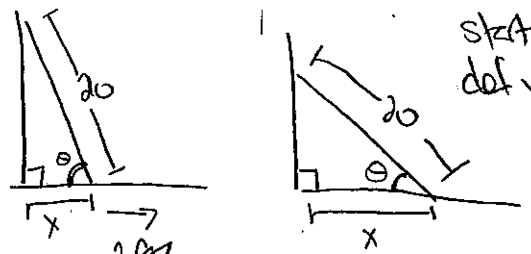
So (+1) $\boxed{(y - 1) = \frac{-3}{4}(x - 2)}$

7. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (WordProblemsWks #11) Consider a ladder 20ft long leaning against a vertical wall. If the bottom of the ladder slides away front he wall at a rate of 2ft/s, how fast is the angle between the latter and the ground changing when the bottom of the latter is 8ft from the wall?

(b) (WebHW11 #9) Two cars start moving from the same point. One travels south at 64 mi/h and the other travels west at 48 mi/h. At what rate is the distance between the cars increasing three hours later?

a)



start (+.5)
def variables (+.5)

HAVE $dx/dt = 2$ ft/s WANT $\underbrace{\left.d\theta/dt\right|_{x=8}}_{(+.5)}$

find relation between x & θ ...
Sohcahtoa $\Rightarrow$ $\cos\theta = \frac{x}{20}$ (+1)

$\frac{d}{dt}(\cos\theta) = \frac{d}{dt}\left(\frac{x}{20}\right)$

$\underbrace{(-\sin\theta)}_{(+.5)}\underbrace{\left(\frac{d\theta}{dt}\right)}_{(+.5)} = \underbrace{\frac{1}{20}\left(\frac{dx}{dt}\right)}_{(+.5)}$

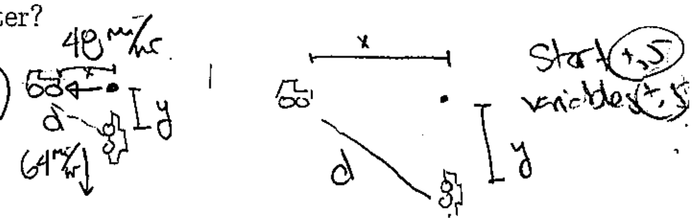
$\Rightarrow d\theta/dt = \dfrac{\frac{dx}{dt}\frac{1}{20}}{-\sin\theta}$

(+1) Need $\sin\theta$ when $x=8$

$\sin\theta = \text{opp}/\text{hyp}$
$\text{opp}^2 + 8^2 = 20^2$
$\Rightarrow \text{opp} = \sqrt{400-64} = \sqrt{336}$

So $\left.d\theta/dt\right|_{x=8} = \dfrac{(2)\frac{1}{20}}{\frac{\sqrt{336}}{20}} = \dfrac{2}{\sqrt{336}}$

b)



Start (+.5)
variables (+.5)

HAVE $dx/dt = 48$ mi/hr $dy/dt = 64$ mi/hr
WANT $\underbrace{\left.dd/dt\right|_{t=3}}_{(+.5)}$

find a relation between x, y & d ... (+1)
Pythagoras? $x^2 + y^2 = d^2$

$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(d^2)$
$2x\,dx/dt + 2y\,dy/dt = 2d\,dd/dt$ (+1.5)

$\Rightarrow dd/dt = \dfrac{x\,dx/dt + y\,dy/dt}{d}$

(+1) need $d, x,$ and y when $t=3$
$x = 48 \cdot 3 = 144$ mi
$y = 64 \cdot 3 = 192$ mi
$\Rightarrow d^2 = 144^2 + 192^2$ so $d = \sqrt{144^2 + 192^2}$

So $\left.dd/dt\right|_{t=3} = \dfrac{(144)(48) + (64)(192)}{\sqrt{144^2 + 192^2}}$