

3. For each f defined below, find $f'(x)$.

$$f(x) = x^4 + 2e^x$$

$$f(x) = e^{x+4} - 7e^2$$

$$f(x) = \frac{e^x + 7}{e}$$

$$\begin{aligned} f'(x) &= [x^4 + 2e^x]' \\ &= [x^4]' + [2e^x]' \\ &= 4x^3 + 2[e^x]' \\ &= 4x^3 + 2e^x \end{aligned}$$

$$\begin{aligned} \frac{d}{dx} f(x) &= \frac{d}{dx} [e^{x+4} - 7e^2] \\ &= \frac{d}{dx} [e^4 e^x - 7e^2] \\ &= \frac{d}{dx} [e^4 e^x] - \frac{d}{dx} [7e^2] \\ &= e^4 \frac{d}{dx} [e^x] - \frac{d}{dx} [7e^2 x^0] \\ &\quad \text{constant} \quad \quad \quad \text{constant} \\ &= e^4 e^x - 0 = e^{4+x} \end{aligned}$$

$$\begin{aligned} f'(x) &= \left[\frac{e^x + 7}{e} \right]' \\ &= \left[\frac{1}{e} e^x + \frac{7}{e} \right]' \\ &= \left[\frac{1}{e} e^x \right]' + \left[\frac{7}{e} x^0 \right]' \\ &= \frac{1}{e} [e^x]' + \frac{7}{e} [x^0]' \\ &\quad \text{constant} \quad \quad \quad \text{constant} \\ &= \frac{1}{e} e^x - 0 = e^{x-1} \end{aligned}$$

4. Consider $\alpha(x) = x^4 + 2e^x$.

(a) Find the equation of the line tangent to the graph of α at the point $(0, 2)$.

Looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$
 $m = \text{slope of line} = \alpha'(0)$ from #3 we know $\alpha'(x) = 4x^3 + 2e^x$
 tangent to α when $x=0$ so $\alpha'(0) = 4 \cdot 0^3 + 2 \cdot e^0 = 0 + 2 \cdot 1 = 2$
 $\Rightarrow m = 2$. The line passes thru $(0, 2)$ so $2 = 2 \cdot (0) + b \Rightarrow b = 2$
 $\Rightarrow y = 2x + 2$ or $y - 2 = 2(x - 0)$

(b) Find the line normal to the line you found in part (a) that also passes through the point $(0, 2)$.

Recall two lines are normal if their slopes are perpendicular
 The line in (a) has slope 2 so a perpendicular line has slope $-\frac{1}{2}$.
 The line passes thru $(0, 2)$ so $2 = -\frac{1}{2}(0) + b \Rightarrow b = 2 \Rightarrow y = -\frac{1}{2}x + 2$

5. At what point on the curve of $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?

Let $\beta(x) = 1 + 2e^x - 3x$. We want to find x so that
 the slope of the line = slope of $3x - y = 5$ note $3x - y = 5$
 tangent to β at x $3x - 5 = y$
 — or —
 $\beta'(x) = 3$ we can compute $\beta'(x) = [1 + 2e^x - 3x]'$
 $2e^x - 3 = 3 \Rightarrow 2e^x = 6 \Rightarrow e^x = 3 \Rightarrow \ln e^x = \ln 3$
 $2e^x = 6 \Rightarrow e^x = 3 \Rightarrow x = \ln 3$