Key

EXAM 1 TMath 124 Winter 2014

/50

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let $f$ be a function and $x$ and $y$ be positive numbers.

   T F $(x+1)^{\frac{3}{2}} = \frac{1}{(x+1)^3}$.   $(x+1)^{3/2} = \sqrt{(x+1)^3}$   $(x+1)^{-3} = \frac{1}{(x+1)^3}$

   T F If $\lim_{h\to 0}[f(h)] = 5$ and $\lim_{h\to o} g(h) = 0$, then $\lim_{h\to 0} \frac{f(h)}{g(h)}$ does not exist. Consider $\lim_{h\to 0} \frac{h(h+5)}{h}$

   T F If $f$ is continuous and $\lim_{x\to -2} f(x) = 5.2$, then $f(-2) = 5.2$.

   T F If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$. If $f$ is differentiable at a then $f$ is con't at a

   T F The semicircle is the graph of a differentiable function. problems at the ends? (vertical tangent lines)

   T F $\frac{d}{dx}(e^x) = xe^{x-1}$   $\frac{d}{dx}(e^x) = e^x$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

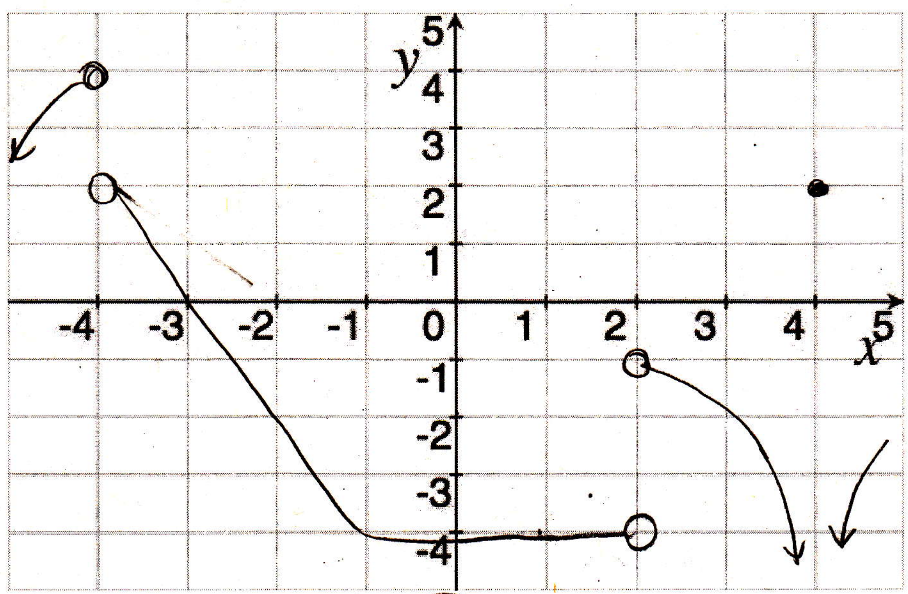
2. [4] (PracticeExam #2) Sketch the graph of an example function $f$ that satisfies the following conditions: +.5

   (a) $f$ is discontinuous at $x=-4$ and $x=2$ +1

   (b) $\lim_{x\to 4} f(x) = -\infty$ +1

   (c) $f(4) = 2$ +.5

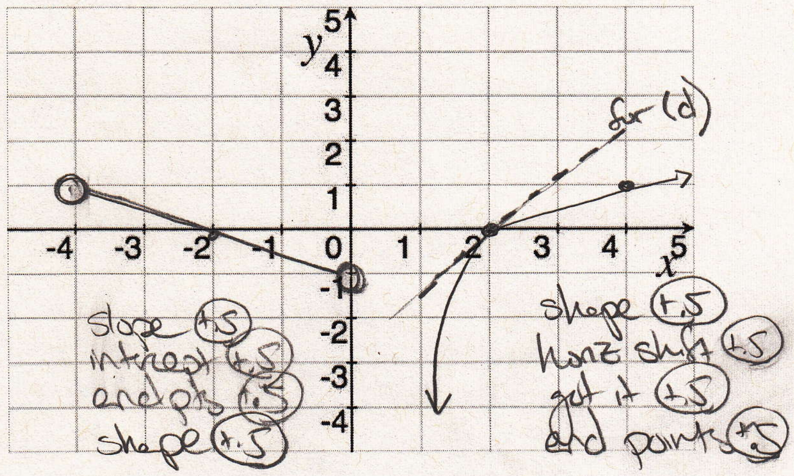
   (d) $f'(-2) < 0$ +1




note: there are many right answers!

3. Let

$$g(x) = \begin{cases} -\frac{1}{2}x - 1 & \text{if } -4 < x < 0. \\ \log_3(x-1) & \text{if } 0 \le x \end{cases}$$




(a) [4] (Quiz1 #1) *Carefully* graph $g$ below.

(b) [1] (§2.2 #5) Estimate $\lim_{x\to 0^-} g(x)$

$-1$

(c) [2] (§2.6 #4) Estimate $\lim_{x\to 1^+} g(x)$

$-\infty$

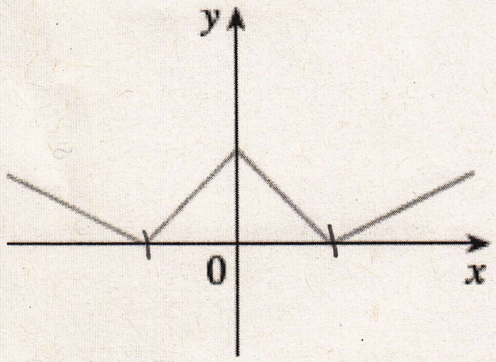
(d) [2] (WebHW5 #6) Estimate $g'(2)$

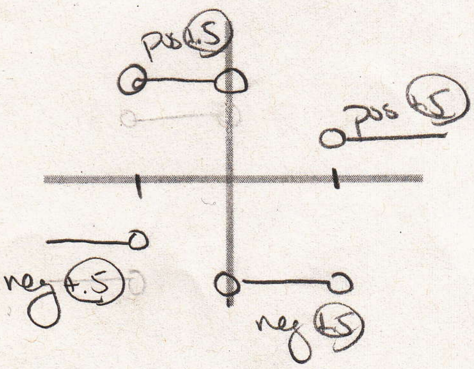
1.5

(e) [2] (§2.3 #2) Estimate $\lim_{x\to -2} [6g(x) - 3]$ $= 6 \lim_{x\to -2} g(x) - 3$ (+1)

$= 6(0) - 3 = -3$ (+1)

4. [4] (WebHW6 #4) Consider the function $m$ graphed on the left. Sketch $m'$.




(+1) looking @ slope

endpoints (+1)

answers to following are done algebraically but numeric or graphical answers/work suffice

5. [12] (§2.3 #3, WebW4 #4, limit laws wks #4, & PracticeExam #4) Find the limit or explain why it does not exist.

$$\lim_{x\to -1}(2x^2 - x + 1)$$

notation (+.5)
algebra (+.5)
limit laws (+1)

$$= \lim_{x\to -1} 2x^2 - \lim_{x\to -1} x + \lim_{x\to -1} 1$$

arithmetic (+.5)

$$= 2\lim_{x\to -1} x^2 - (-1) + 1$$

$$= 2(-1)^2 + 1 + 1 = 2+1+1 = 4 \} (+.5)$$

or

Hector's Thm (+1.5)

notation (+.5)
arithmetic (+.5)

$$2(-1)^2 - (-1) + 1$$

$$= 4 \} (+.5)$$

$$\lim_{x\to 9}\frac{16+\sqrt{x}}{\sqrt{16+x}}$$

notation (+.5)
algebra (+.5)
limit law (+1)
arithmetic (+.5)

$$= \frac{\lim_{x\to 9} 16+\sqrt{x}}{\lim_{x\to 9}\sqrt{16+x}}$$

$$= \frac{\lim_{x\to 9} 16 + \lim_{x\to 9}\sqrt{x}}{\sqrt{\lim_{x\to 9}(16+x)}}$$

$$= \frac{16+\sqrt{9}}{\sqrt{16+9}} = \frac{16+3}{\sqrt{25}}$$

$$= 19/5 = 3.8 \} (+.5)$$

$$\lim_{h\to 0}\frac{3-(2+h)^2+1}{h}$$

algebra (+1)
notation (+.5)
limit laws (+1)

$$= \lim_{h\to 0}\frac{3-(2+4h+h^2)+1}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{2}-4h-h^2+\cancel{2}}{h}$$

$$= \lim_{h\to 0}\frac{-4h-h^2}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(-4-h)}{\cancel{h}}$$

$$= \lim_{h\to 0}(-4-h)$$

$$= -4-0 = -4 \} (+.5)$$

$$\lim_{x\to\infty}\frac{\sin x}{x^3}$$

notation (+.5)
logic (+1)

Note $-1 \le \sin x \le 1 \} (+.5)$

and as $x$ gets larger + larger $1/x^3 > 0$ so

$$-1\left(\frac{1}{x^3}\right) \le \left(\frac{1}{x^3}\right)\sin x \le \left(\frac{1}{x^3}\right)1$$

or

$$\frac{-1}{x^3} \le \frac{\sin x}{x^3} \le \frac{1}{x^3} \quad (+.5)$$

notice $\lim_{x\to\infty} \frac{-1}{x^3} = 0$ by Big Little

and $\lim_{x\to\infty} \frac{1}{x^3} = 0$ by Big-Little

By the squeeze Theorem $\} (+.5)$

$$\lim_{x\to\infty}\frac{\sin x}{x^3} = 0$$

6. [5] (poly & exp wks #4) Let $f(x) = x^3 - 3e^x$. Find the equation for the line tangent to the graph of $f$, when $x = 0$.

Looking for $y = mx + b$ } +.5

+1 { $m$ = slope of line tangent to $f$ when $x = 0$

$= f'(0)$

+.5 finding $f'(x)$

$\frac{d}{dx}(x^3 - 3e^x) = \frac{d}{dx}(x^3) - \frac{d}{dx}(3e^x)$ +.5 +.5

$= 3x^2 - 3e^x$

power rule b/c $\frac{d}{dx}(e^x) = e^x$

plug in +.5 { $= 3(0)^2 - 3 \cdot e^0$

$= 0 - 3 \cdot 1 = -3$

the line passes thru $(0, f(0)) = (0, 0^3 - 3 \cdot e^0) = (0, -3)$ +.5 +.5 got it.

so

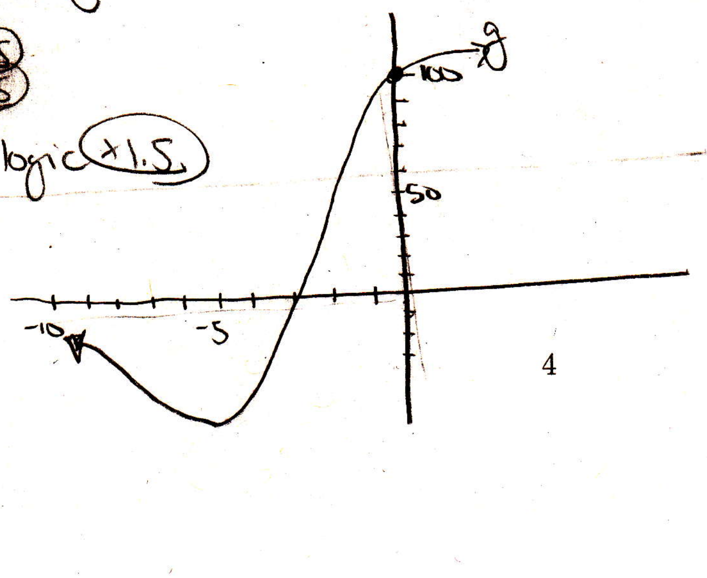
$-3 = -3(0) + b$ } plug in +.5 $\Rightarrow b = -3$

Thus $y = -3x - 3$ or $y - -3 = -3(x - 0)$

7. [3] (§2.5 #50) Assume that $g$ is continuous, $\lim_{x \to -\infty} g(x) = -3$, and $g(0) = 100$. Is the value of $g(-5)$ between $-3$ and $100$? *Justify* your answer.

Maybe not. For example consider

start +.5
def of con't +.5
def of lim +.5
counter ex / logic +1.5




where the graph of $g$ is con't.

$g(0) = 100$
and $\lim_{x \to -\infty} g(x) = -3$

BUT $g(-5)$ is less than $-3$.

8. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§2.6 #63) Under certain assumptions the velocity $v(t)$ of a falling raindrop at time $t$ is:

$$v(t) = v^*(1 - e^{\frac{-gt}{v^*}})$$

where $g$ is the acceleration due to gravity (9.8 m/s$^2$).

i. [3] Find $\lim_{t\to\infty} v(t)$.

ii. [2] Interpret the answer given in (i) as a scientist and explain what $v^*$ is in everyday language.

(b) A rock thrown upwards on planet Mars with velocity $15\frac{\text{m}}{\text{s}}$ has a height $h(t) = 15t - 1.86t^2$ meters $t$ seconds later.

i. [2] (Story wks #6) Find the velocity of the rock after 2 seconds.

ii. [3] (Story wks 6b) *When* does the rock have a velocity of $5\frac{\text{m}}{\text{s}}$?

(a)

i) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty} v^*(1-e^{\frac{-gt}{v^*}})$

$= \lim_{t\to\infty} v^*\left(1 - \frac{1}{e^{gt/v^*}}\right)$

start (+.5) limit prop (+.5) logic (+.5)

notice as $t\to\infty$ $gt/v^* \to \infty$

exp (+.5) so $\frac{1}{e^{gt/v^*}}$ is "$\frac{1}{\text{Big}}$" or little (+.5)

ie $\frac{1}{e^{gt/v^*}} \to 0$

So the limit is $v^*(1-0) = v^*$ (+.5)

ii) $v^*$ (+.5) is the terminal velocity (+1.5)
(the maximum velocity a raindrop can reach)

(b)

i) velocity = $\frac{d}{dt}(h(t))$ (+.5)
$= \frac{d}{dt}(15t - 1.86t^2)$
$= 15 - 3.72t$
(+.5) (+.5)

velocity(2) $= 15 - 3.72(2)$ (+.5)
$= 15 - 7.44$ arithmetic/alg (+.5)
$= 7.56$ start (+.5)

ii) ie find $t$ so that
$5 = f'(t)$ (+1)
$5 = 15 - 3.72t$ alg (+1)
$\Rightarrow -10 = -3.72t$
$\Rightarrow t = 10/3.72$ s
$\approx 2.6882$ s (+.5)