Key

TMATH 124pm: Quiz 4

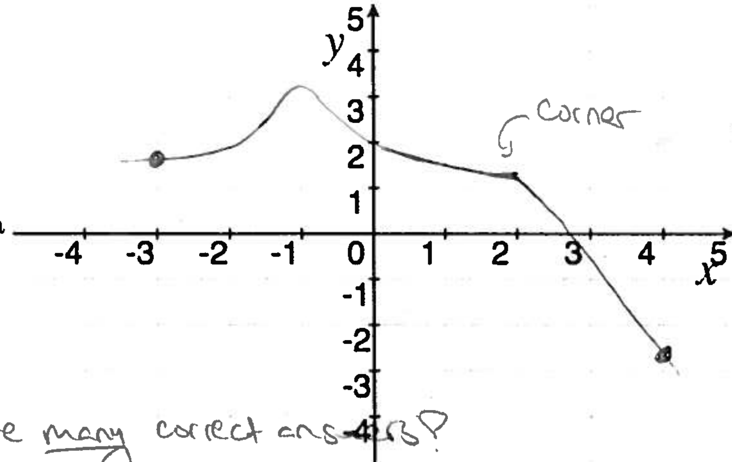
Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [3] (extreme wks #1) Sketch the graph of an example function f that satisfies all of the following conditions:

   (+1) (a) f is continuous on the interval $[-3, 4]$

   (+1) (b) $x = -1$ and $x = 2$ are critical numbers.

   (+1) (c) the only local extrema (maximums or minimums) occur when $x = -1$ or $x = 4$.



note there are many correct answers?

2. [2] (Lecture /28) Given that the function g is:

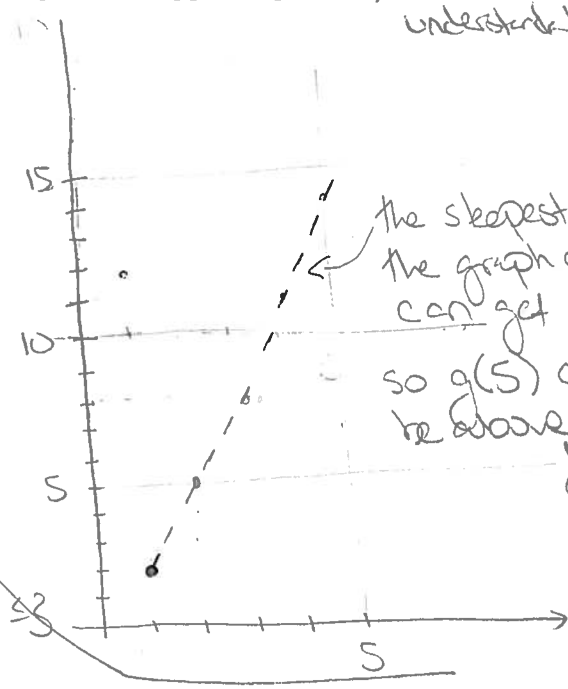
   - continuous on $[0, \infty)$
   - differentiable on $(0, \infty)$
   - evaluates to 2 when given the input 1, and
   - that $g'(x) \leq 3$ for all x,

   find an upper bound for $g(5)$. Explain yourself or provide supporting work.

graphed reasoning (+.5)
understandable (+1)

note we know (1,2) is on the graph of g

from the mean value theorem we know there exists a c between 1 and 5 so that or



the steepest that the graph of g can get

so g(5) can't be above 14 (+.5)

$$\frac{g(5)-g(1)}{5-1} = g'(c)$$

$$\Rightarrow \frac{g(5)-2}{4} = g'(c) \quad \text{since } g'(c) \leq 3$$

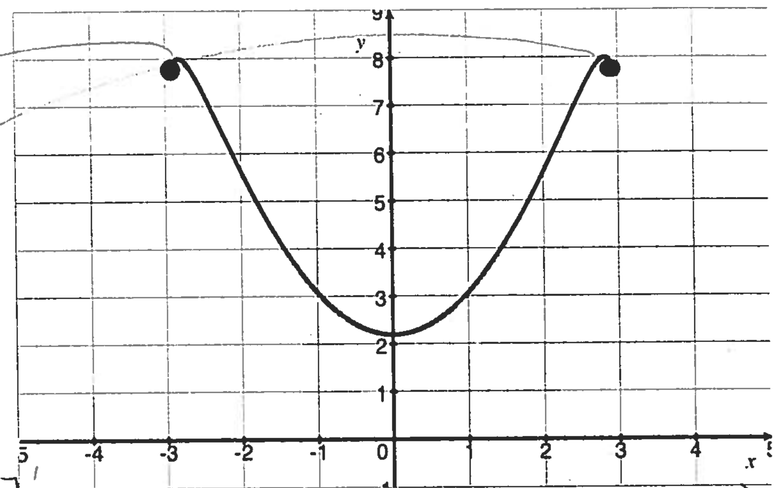
$$\Rightarrow \frac{g(5)-2}{4} \leq 3 \Rightarrow g(5)-2 \leq 12 \Rightarrow g(5) \leq 14$$

3. Consider the graph of $h(x) = \ln(e^{(x^2)}(9 - x^2)$ on the interval $[-2.9, 2.9]$.

(a) [1] (§4.1 #66) Use the graph of h shown to the right, to *estimate* the x values of all absolute maximums.

-2.8 and 2.8

(+.5) (+.5)



(b) [2] (§3.6 #8) Find $h'(x)$ and simplify.

up of ln (+.5)
(correct (+.5)
er of x^2 (+.5)
er of $\ln(9-x^2)$ (+.5)

or

er of ln (+.5)
product rule (+.5)
2 chains (+1)

$$h'(x) = [\ln(e^{x^2}(9-x^2))]'$$
$$= [\ln e^{x^2} + \ln(9-x^2)]'$$
$$= [x^2 + \ln(9-x^2)]'$$
$$= [x^2]' + [\ln(9-x^2)]$$
$$= 2x + -2x\left(\frac{1}{9-x^2}\right) \Big\} \star$$

The derivative of $\ln(9-x^2)$

outside function $= \ln u$ outside$' = 1/u$

inside function $= 9-x^2$ inside$' = -2x$

outside$'$(inside)$\cdot$inside$'$

$= \frac{1}{9-x^2} \cdot -2x$

(c) [2] (§4.1 #66) Calculate the *exact* x-values that correspond to the absolute maximums of h.

looking for CP (+.5)

We need to find the critical points & see which ones correspond to the -2.9 & 2.8 estimates above...

CP when: $h'(x)$ DNE or $h'(x) = 0$] (+.5)

@ endpoints

$\Rightarrow x = -2.9$

$x = 2.9$

$$2x - \frac{2x}{9-x^2} = 0$$

algebra (+1)

$$\Rightarrow 2x = \frac{2x}{9-x^2}$$
$$\Rightarrow 2x(9-x^2) = 2x$$
$$18x - 2x^3 = \cancel{2x}$$
$$-2x \quad \cancel{-2x}$$
$$16x - 2x^3 = 0$$
$$2x(8-x^2) = 0$$
$$\Rightarrow x = 0 \quad \text{or} \quad x = \pm\sqrt{8}$$

So the absolute max's occur when

$x = \pm\sqrt{8}$

note when $x=0$ we have an abs min