TMATH 124pm: Quiz 3

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [5] (product & quotient wks #4) Let $f(x) = 5^x(1+x^2)$

   (a) Find the slope of the line tangent to the graph of f when $x = 2$.

notation (+.5)

slope of line tangent to the graph f when x=2 $= f'(2)$ } (+.5)

finding $f'(x)$

$f'(x) = 5^x(1+x^2)' + (1+x^2)(5^x)'$ } (+1)

$= 5^x(2x) + (1+x^2)\cdot 5^x \ln 5$ (+.5)

$= 5^x \cdot 2x + (1+x^2)\cdot 5^x \ln 5$

$f'(2) = 5^2 \cdot 2(2) + (1+(2)^2)\cdot 5^2 \ln 5$ } (+.5)

$= 25\cdot 4 + 5\cdot 25\cdot \ln 5$

$= 100 + 125\ln 5$

$\approx$

   (b) Find where the function f has a horizontal tangent line.

find x so that

slope of line tangent to f at x = slope of a horizontal line

$\Rightarrow$ (+.5) } $f'(x) = 0$

from part (a)

(+.5) } $5^x \cdot 2x + (1+x^2)5^x \ln 5 = 0$

$\Rightarrow 5^x(2x + (1+x^2)\ln 5) = 0$

$\Rightarrow 5^x = 0$ or $2x + (1+x^2)\ln 5 = 0$

never happens

Algebra (+1)

$\Rightarrow 2x + \ln 5 + (\ln 5)x^2 = 0$

$(\ln 5)x^2 + 2x + \ln 5 = 0$

solve the quadratic?

[quadratic formula or complete the square]

$x = \dfrac{-2 \pm \sqrt{4 - 4(\ln 5)(\ln 5)}}{2\cdot \ln 5}$

$\approx$

2. [2] (§3.3 #43) Find $\lim_{x\to 0} \frac{\sin(3x)}{5x^3 - 4x}$ $= \lim_{x\to 0} \frac{\sin(3x)}{x(5x^2-4)}$

use $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (+.5)

algebra (+.5)

limit laws (+.5)

$= \lim_{x\to 0} \underbrace{\frac{3}{3}}_{(+.5)} \frac{\sin 3x}{x} \cdot \frac{1}{5x^2-4}$

$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{3}{5x^2-4}$

$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0} \frac{3}{5x^2-4} = 1 \cdot \frac{3}{5\cdot 0^2 - 4} = -3/4$

3. [3] (WebHW9 #9) Let $g(x) = \cos\left(\frac{e^x}{1+x^4}\right)$. Find $g'(x)$ and *simplify*.

$\underbrace{\cos\left(\frac{e^x}{1+x^4}\right)}$

(+.5) chain rule

$f(u) = \cos u$ $\quad f'(u) = -\sin u$ (+.5)

$g(x) = \frac{e^x}{1+x^4}$ $\quad g'(x) = \overbrace{\frac{(1+x^4)(e^x)' - e^x(1+x^4)'}{(1+x^4)^2}}^{(+.5)} = \frac{(1+x^4)e^x - e^x 4x^3}{(1+x^4)^2}$ (+.5) (+.5)

note $f(g(x)) = f\left(\frac{e^x}{1+x^4}\right) = \cos\left(\frac{e^x}{1+x^4}\right)$ ✓

$\left[\cos\left(\frac{e^x}{1+x^4}\right)\right]' = f'(g(x))\cdot g'(x)$

$= f'\left(\frac{e^x}{1+x^4}\right) \cdot \left[\frac{(1+x^4)e^x - e^x 4x^3}{(1+x^4)^2}\right]$

used chain rule (+.5)

$= -\left[\sin\left(\frac{e^x}{1+x^4}\right)\right] \cdot \left[\frac{e^x[(1+x^4) - 4x^3]}{(1+x^4)^2}\right]$

$= -\left[\sin\left(\frac{e^x}{1+x^4}\right)\right] \cdot \left[\frac{e^x(x^4 - 4x^3 + 1)}{(1+x^4)^2}\right]$