Key

TMATH 124pm: Quiz 2

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. (§2.7 #4) Let $f(x) = x - x^3$

$f'(x) = 1 - 3x^2$ $\quad f'(3) = 1 - 3(3)^2 = 1 - 27 = -26$

(a) [2] Find the slope of the line tangent to the graph of f when $x = 3$.

(+.5) $= f'(3)$

slope of line tangent to f when $x = 3$

$$= \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{x - x^3 - (3 - 3^3)}{x - 3} = \lim_{x \to 3} \frac{-x^3 + x + 24}{x - 3}$$

(+.5)

$$= \lim_{x \to 3} \frac{(x-3)(-x^2 - 3x - 9)}{x - 3}$$

(+.5)

$$= \lim_{x \to 3} (-x^2 - 3x - 9)$$

(+.5)

$$= -(3)^2 - 3(3) - 9 = -26$$

(+.5)

$-x^2 - 3x - 9$

$3 \mid -x^3 + 0x^2 + x + 24$

$-(-x^3 + 3x^2)$

$-3x^2 + x + 24$

$-(-3x^2 + 9x)$

$-8x + 24$

$-8x + 24$

or

(+.5) $= f'(3)$

$$= \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{[(3+h) - (3+h)^3] - [3 - 3^3]}{h}$$

(+.5)

$$= \lim_{h \to 0} \frac{3 + h - 27 - 3 \cdot 9h - 3 \cdot 3h^2 - h^3 + 24}{h}$$

(+.5)

$$= \lim_{h \to 0} \frac{-26h - 9h^2 - h^3}{h}$$

(+.5)

$$= \lim_{h \to 0} \frac{h(-26 - 9h - h^2)}{h}$$

(+.5)

$$= -26 - 9 \cdot 0 - 0^2 = -26$$

(b) [2] Find the equation of the line tangent to the graph of f when $x = 3$.

looking for $y = mx + b$ (+.5)

m = slope of line tangent to f when $x = 3$ $= f'(3) = -26$ (from a) (+.5)

$y - y_1 = m(x - x_1)$

$y - f(a) = m(x - a)$

line passes thru $(3, f(3)) = (3, -24)$ (+.5)

So $-24 = -26(3) + b$

$\Rightarrow b = -24 + 3 \cdot 26$

$b = -24 + 78$

$b = 54$ (+.5)

$\rightarrow y = -26x + 54$

2. [2] (WebHW4 #11) Let $g(x) = \dfrac{x+4}{x^2-5}$. Find $\lim_{x\to-\infty} g(x)$.

$\lim_{x\to\infty} \dfrac{x+4}{x^2-5} \cdot \dfrac{(1/x^2)}{(1/x^2)}$ } +.5

$= \lim_{x\to\infty} \dfrac{x/x^2 + 4/x^2}{x^2/x^2 - 5/x^2}$

$= \lim_{x\to\infty} \dfrac{1/x + 4/x^2 \to 0}{1 - 5/x^2 \to 0} = \dfrac{0}{1} = 0$ +.5

alg +.5 or Big-Little +.5

denominator is growing faster than numerator so $\frac{1}{\text{Big}}$ = little so 0

3. [4] (Con't Wks #6) Sketch a graph of a function α that satisfies *all* of the following:

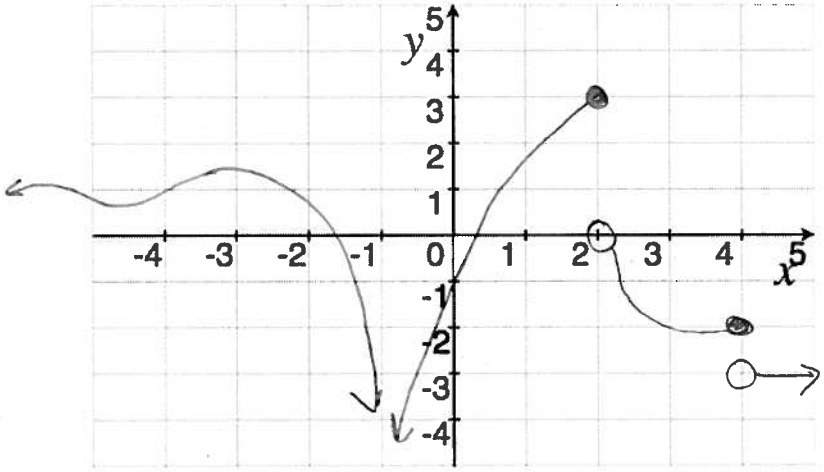
$\lim_{x\to 2^+} \alpha(x) = 0$ +.5

$\lim_{x\to -1} \alpha(x) = -\infty$ +1

$\alpha(2) = 3$ +.5

α is discontinuous when $x = 4$ +1

$\lim_{x\to-\infty} \alpha(x) = 1$ +1



one of many correct answers.