EXAM 2 # TMath 124am Winter 2013

Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be a functions and let x and y be positive numbers.

T F $\sin(3x) = 3\sin(x)$

T F $\frac{4x^2-3}{-x} = -4x + \frac{3}{x}$ $\frac{4x^2-3}{-x} = \frac{4x^2}{-x} + \frac{-3}{-x} = \frac{4x}{-1} + \frac{3}{x} = -4x + 3/x$

T F $(f \cdot g)'(x) = f'(x)g'(x)$ $(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$ product rule

T F $\frac{d}{dt}(x^2) = 2x$ $\frac{d}{dt}(x^2) = 2x \frac{dx}{dt}$ by chain rule

T F $\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{dy}{dx}(x^{-1}) - x^{-2}y$ $\frac{d}{dx}(yx^{-1}) = \frac{d}{dx}(y) \cdot x^{-1} + y \frac{d}{dx}(x^{-1})$ $= \frac{dy}{dx} \cdot x^{-1} - yx^{-2}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

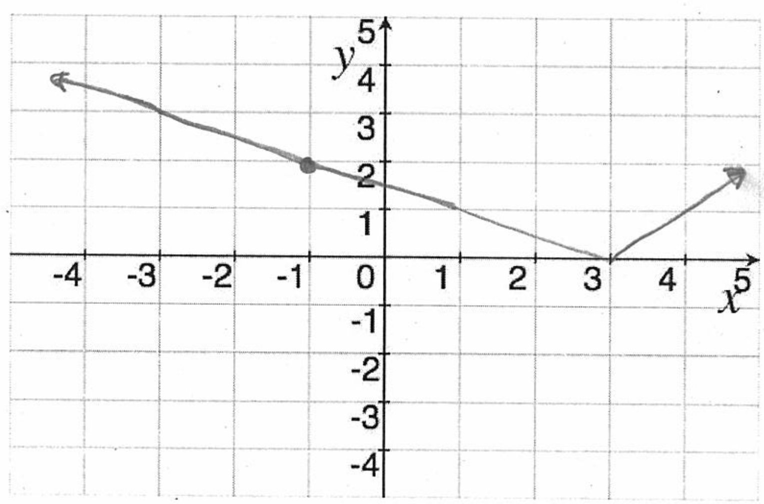
2. [4] (practice exam2 #2) Sketch the graph of an example function f that satisfies the following conditions:

(a) f is not differentiable when $x = 3$ (+1)

(b) f is continuous when $x = 3$ (+1)

(c) $f(-1) = 2$ (+1)

(d) $f'(-1) = -\frac{1}{2}$ (+1)



note: there are many correct answers.

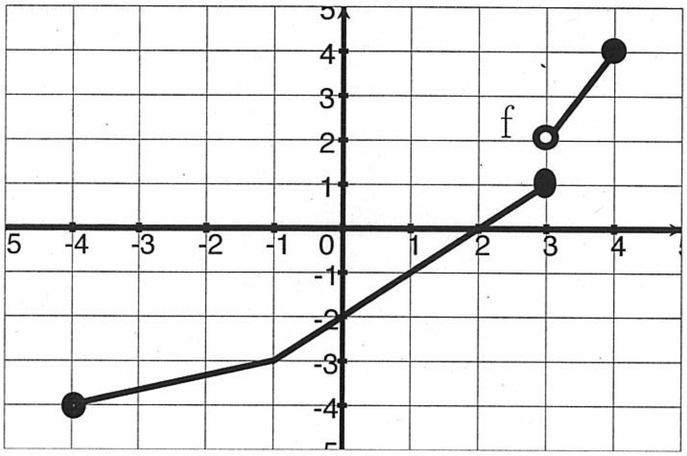
3. [4] (practice exam2 #3) Find a formula for the function f you drew in problem (2).

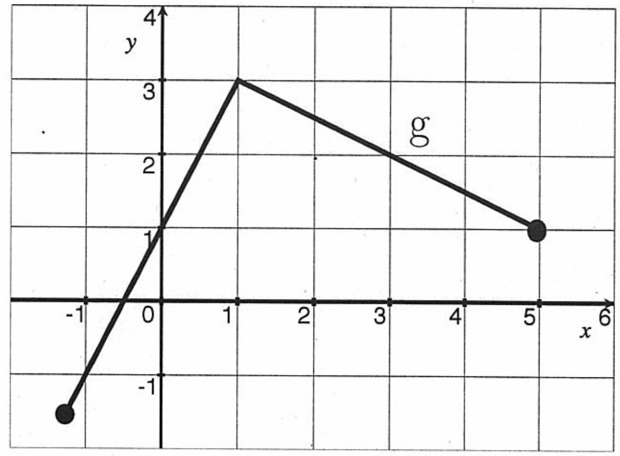
$$f(x) = \begin{cases} -\frac{1}{2}x + \frac{3}{2} & \text{if } x \le 3 \\ x - 3 & \text{if } 3 < x \end{cases}$$

note passes thru $(-1, 2)$
so $2 = -\frac{1}{2}(-1) + b$
$\Rightarrow b = 2 - \frac{1}{2} = \frac{3}{2}$

(+1) for each condition in 2

4. Let the graph of f and g be those shown below:





Estimate the following (if they exist)

[2] (WebHW8 #7)

$\frac{d}{dx}(f \cdot g)|_{x=1} = f'(1)g(1) + f(1)g'(1)$ +1

product +.5 plug in 1 +.5

$= (1)(3) + (-1)$ DNE

f(1) + g(1) +.5
f'(1) + g'(1) +.5

b/c g has a corner when x = 1
the graph of f·g has a corner

+2 $\Rightarrow (f \cdot g)'(1)$ Does not exist

[2] (WebHW8 #8)

tried quotient +.5
used quotient +.5
plug in 0 ... +.5

$\left(\frac{f}{g}\right)'(0) = \frac{g(0)f'(0) - f(0)g'(0)}{[g(0)]^2}$

$= \frac{(1)(1) - (-2)(2)}{(1)^2}$ +1

$= \frac{1+4}{1} = 5$

[2] (§3.2 #44)

If $h(x) = \left(\frac{g(x)}{1+f(x)}\right)$, $h'(2)$

$h'(x) = \frac{(1+f(x))g'(x) - g(x)(1+f(x))'}{[1+f(x)]^2}$ +.5

$= \frac{(1+f(x))g'(x) - g(x)f'(x)}{[1+f(x)]^2}$ +.5

$h'(2) = \frac{(1+0)(-\frac{1}{2}) - (\frac{5}{2})(1)}{(1+0)^2}$ +1

$= \frac{(-\frac{1}{2}) - \frac{5}{2}}{1^2} = \frac{-3}{1^2} = -3$

[2] (§3.4 #65)

$\frac{d}{dx}(f \circ g)|_{x=0}$

$f'(g(0))\,g'(0)$ +1

$f'(1) \cdot 2$

$(1) \cdot 2$ +1

$= 2$

5. Find $\frac{dy}{dx}$ of following. Simplify.

notation (+.5)

[2] (WebHW9 #8)

$y = x^7 + 7^x + 7^7$

$$\frac{dy}{dx} = \frac{d}{dx}(x^7 + 7^x + 7^7)$$
$$= \frac{d}{dx}(x^7) + \frac{d}{dx}(7^x) + \frac{d}{dx}(7^7) \Big\} (+.5)$$
$$= \underbrace{7x^6}_{(+.5)} + \underbrace{7^x \ln 7}_{(+.5)} + \underbrace{0}_{(+.5)}$$

[3] (§3.2 #2)

$y = \frac{x^4 - 5x^2 + \sqrt{x}}{x^2}$ $= \frac{x^4}{x^2} - \frac{5x^2}{x^2} + \frac{x^{1/2}}{x^2}$

$$= x^2 - 5 + x^{-3/2} \Big\} (+1)$$
$$\frac{dy}{dx} = \frac{d}{dx}(x^2) - \frac{d}{dx}(5) + \frac{d}{dx}(x^{-3/2})$$
$$= 2x - 0 + -\tfrac{3}{2}x^{-5/2}$$
(+.5) (+.5) (+.5)
$$= 2x - \tfrac{3}{2}x^{-5/2}$$

or

$$\frac{dy}{dx} = \frac{x^2 \frac{d}{dx}(x^4 - 5x^2 + x^{1/2}) - (x^4 - 5x^2 + x^{1/2})\frac{d}{dx}(x^2)}{(x^2)^2} \quad (+1)$$
$$(+1) \Big\{ = \frac{x^2[4x^3 - 10x + \tfrac{1}{2}x^{-1/2}] - (x^4 - 5x^2 + x^{1/2})2x}{x^4}$$

simplify (+1) ick...

notation (+.5)

[4] (implicit wks)

$y\cos(x) = x^2 + y^2$

$$\frac{d}{dx}(y\cos x) = \frac{d}{dx}(x^2 + y^2)$$
$$y\frac{d}{dx}(\cos x) + \frac{d}{dx}(y)\cos x = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$
product (+.5)
$$y\underbrace{(-\sin x)}_{(+.5)} + \underbrace{\frac{dy}{dx}}_{(+.5)}\cos x = \underbrace{2x}_{(+.5)} + \underbrace{2y\frac{dy}{dx}}_{(+.5)}$$
$$\Rightarrow \frac{dy}{dx}\cos x - 2y\frac{dy}{dx} = 2x + y\sin x$$
$$\Rightarrow \frac{dy}{dx}(\cos x - 2y) = 2x + y\sin x$$
$$\Rightarrow \frac{dy}{dx} = \frac{2x + y\sin x}{\cos x - 2y}$$

solve for $\frac{dy}{dx}$ (+1) if only one y' +.5

[3] (lecture 2/14)

$y = \ln(x)$

recall $y = \ln(x)$ exactly when $e^y = x$ } (+.5)

$$(+1)\ \frac{d}{dx}(e^y) = \frac{d}{dx}(x) \Big\}\ \text{der. prop} \ (+.5)$$
$$e^y \frac{dy}{dx} = 1 \ (+.5)$$
$$\Rightarrow \boxed{\frac{dy}{dx} = 1/e^y}$$ solve for $\frac{dy}{dx}$ (+.5)

b/c $y = \ln x$

$$\frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

so $\frac{dy}{dx} = 1/x$

or

you have it memorized —

$$\frac{d}{dx}(\ln x) = 1/x.$$

6. [3] (§3.3 #44) Find $\lim_{x\to 0} \frac{x\sin(3x)}{3x\sin(4x)}$.

notation (+.5)
algebra (+.5)

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (+.5)

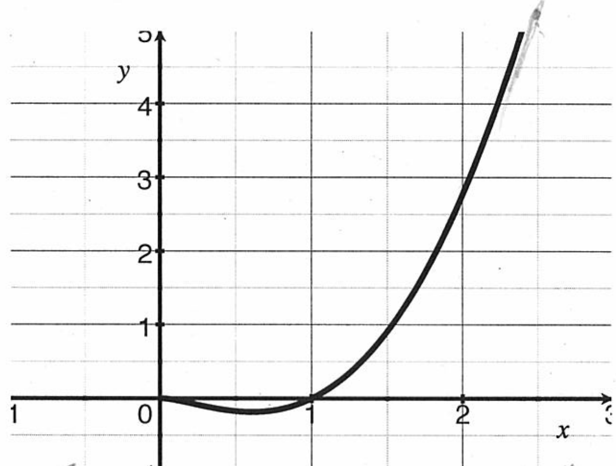
$\lim_{x\to 0} \frac{x}{\sin 4x} \cdot \frac{\sin 3x}{3x}$

$= \lim_{x\to 0} \frac{4}{4}\frac{x}{\sin 4x} \cdot \lim_{x\to 0} \frac{\sin 3x}{3x} \to 1$ (+1)

$= \lim_{x\to 0} \frac{1}{4} \frac{4x}{\sin 4x} = \frac{1}{4} \lim_{x\to 0} \frac{4x}{\sin 4x} \to 1$

$= 1/4$ (+.5)

7. Let $f(x) = x^2\ln(x)$, whose graph is shown to the right.



(a) [3] Find the equation of the line tangent to f when $x = e$.

Looking for $y = mx + b$ (+.5)
m = slope of line tangent to f when $x = e$

(+.5) $= f'(e) = e + 2e\ln e = 3e$ (+.5)

finding $f'(x)$: $[x^2\ln x]'$
$= (x^2)[\ln x]' + [x^2]' \cdot \ln x$ (+.5)
$= x^2 \cdot 1/x + 2x \cdot \ln x = x + 2x\ln x$

(+.5) line passes thru $(e, f(e)) = (e, e^2 \cdot \ln e)$
or (e, e^2)
so $e^2 = 3e(e) + b \Rightarrow b = e^2 - 3e^2 = -2e^2$

(+.5) so $y = 3ex - 2e^2$

(b) [3] (Quiz3 #1b) Find where the function f has a horizontal tangent line.

notation (+.5)

Looking for x when
slope of line tangent to f at x = slope of horz. line

(+1) $f'(x) = 0$

$x + 2x\ln x = 0$ (+.5)
$x(1 + 2\ln x) = 0$
$\cancel{x = 0}$ or $1 + 2\ln x = 0$
outside domain
$\Rightarrow \ln x = -1/2$
$x = e^{-1/2}$

algebra (+1)

(c) [2] Use a linear approximation to estimate the value of $f(2.5)$.

(+1) note that 2.5 is close to e
we can use our line from (a) as a linear approx.

$3e(2.5) - 2e^2 =$ (+1)

8. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) A firetruck, parked 15 ft from the base of a building, extended its ladder 40 ft and leaned it against a tall building. After completing their training exercise, the driver gets into the truck and forgets to bring down the ladder before driving away at 5mi/hr (about 7 ft/sec). For the purposes of this problem, assume the base of the ladder is 8 ft above the ground and 15ft from the building.

i. [3] How fast is the top of the ladder moving down the building when the firetruck/base of the ladder is 20 ft from the base of the building? 35 ft?

ii. [3] What rate is the angle between the base of the ladder and the top of the fire truck decreasing when the ladder is 35 ft from the building?

(b) A particle is moving according to a law of motion $s(t) = 2\sin(\pi t) + 3$.

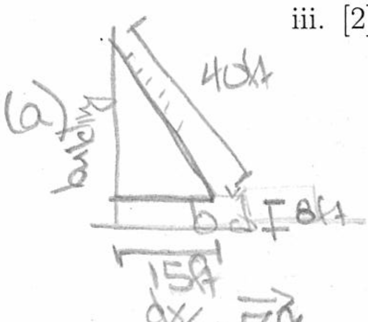
i. [2] Find the velocity of the particle after 3.5 seconds.

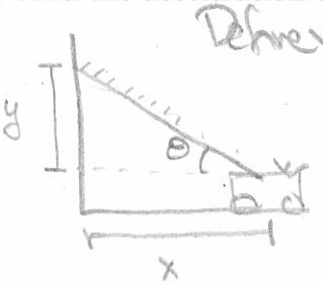
ii. [2] When is the particle moving in the negative direction?

iii. [2] Find the acceleration of the particle as a function of t.

Define variables (+.5)

(a)





$dx/dt = 7$ ft/sec

(+.5) find $dy/dt\big|_{x=20ft}$ and $dy/dt\big|_{x=35ft}$

(+.5) $x^2+y^2=40^2$

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(40^2)$

(+1) $2x\, dx/dt + 2y\, dy/dt = 0$

$dy/dt = \frac{-2x\, dx/dt}{2y}$

(+.5) when $x = 20ft$ $y = \sqrt{40^2-20^2} \approx 34.6$
when $x = 35ft$ $y = \sqrt{40^2-35^2} \approx 5\sqrt{15}$

(+.5) so $dy/dt\big|_{x=20} = \frac{20(7)}{} \approx$ $dy/dt\big|_{x=35} = \frac{20(7)}{} \approx$

ii) find $d\theta/dt\big|_{x=35}$ (+.5)

(+.5) $\cos\theta = \frac{x}{40}$

$\frac{d}{dt}(\cos\theta) = \frac{d}{dt}(\frac{1}{40}x)$

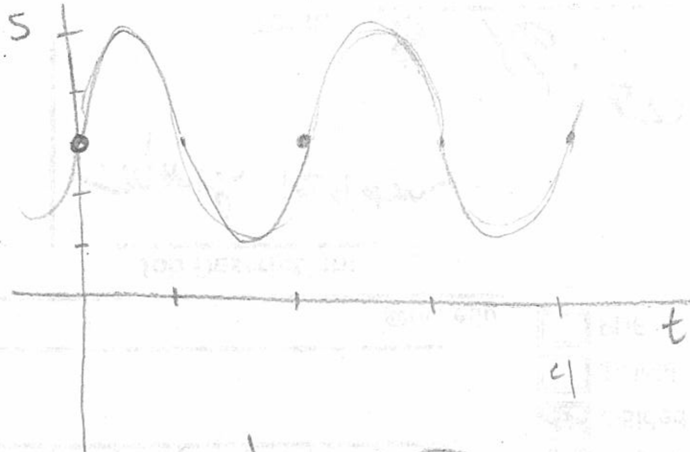
(+1) $(-\sin\theta)\, d\theta/dt = \frac{1}{4}\, dx/dt$

$d\theta/dt = -\frac{1}{4}\, dx/dt\, \frac{1}{\sin\theta}$

(+.5) when $x = 35$ $\cos\theta = 35/40$
$\Rightarrow \theta = \arccos(35/40)$
$\approx$

so $d\theta/dt\big|_{x=35} = -\frac{1}{4}(7)\frac{1}{}$
$\approx$

(b)



$2\sin(\pi t)+3$

amplitude: 2

period: $\frac{2\pi}{\pi}$ or 2

i) velocity $= \frac{ds}{dt}$ } (+.5)

$= \frac{d}{dt}(2\sin(\pi t)+3)$

$= 2(\cos(\pi t))\cdot \pi + 0$ } (+1)

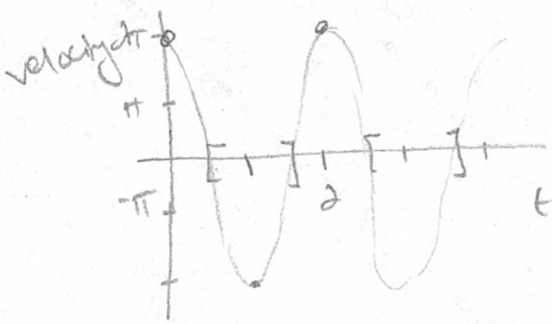
$= 2\pi\cos(\pi t)$

velocity at 3.5 sec is $2\pi\cos(\underbrace{3.5\pi}_{+.5}) \approx 0$

ii) When is velocity negative?

ie when is $2\pi\cos(\pi t) < 0$ } (+.5)

see corresponds w/ slopes +.5



amplitude: 2π

period: $2\pi/\pi$ or 2

between .5 and 1.5 } (+1)

and between 2.5 and 3.5

and between 4.5 and 5.5

repeating } (+.5)

iii) acceleration $= \frac{dv}{dt}$ where v is velocity } (+.5)

$= \frac{d}{dt}(2\pi\cos(\pi t))$

$= 2\pi(-\sin\pi t)\pi$ } (+1)

$= -2\pi^2\sin\pi t$

start (+.5)