Missing neg on #5
~~#9~~
v* is constant

Key

EXAM 1 TMath 124 Winter 2013

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function and x and y be positive numbers.

T F $\sqrt{x+3} = (x+3)^{-2}$. $\sqrt{x+3} = (x+3)^{1/2}$ and $(x+3)^{-2} = \frac{1}{(x+3)^2}$

T F If $\lim_{h\to 0}[f(h)] = 3$ and $\lim_{h\to o} g(h) = 0$, then $\lim_{h\to 0}\frac{f(h)}{g(h)}$ does not exist. consider $\lim_{h\to 0}\frac{h(h+5)}{h} = 5$

T F If f is continuous and $\lim_{x\to -2} f(x) = 5.2$, then $f(-2) = 5.2$.
b/c continuity means $\lim_{x\to a} f(x) = f(a)$.

T F If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.
If f is differentiable at a, then f is continuous.

T F The parabola is the graph of a differentiable function. $\frac{d}{dx}(ax^2+bx+c) = 2ax+b$

T F $\frac{d}{dx}(e^x) = xe^{x-1}$ $\frac{d}{dx}(e^x) = e^x$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

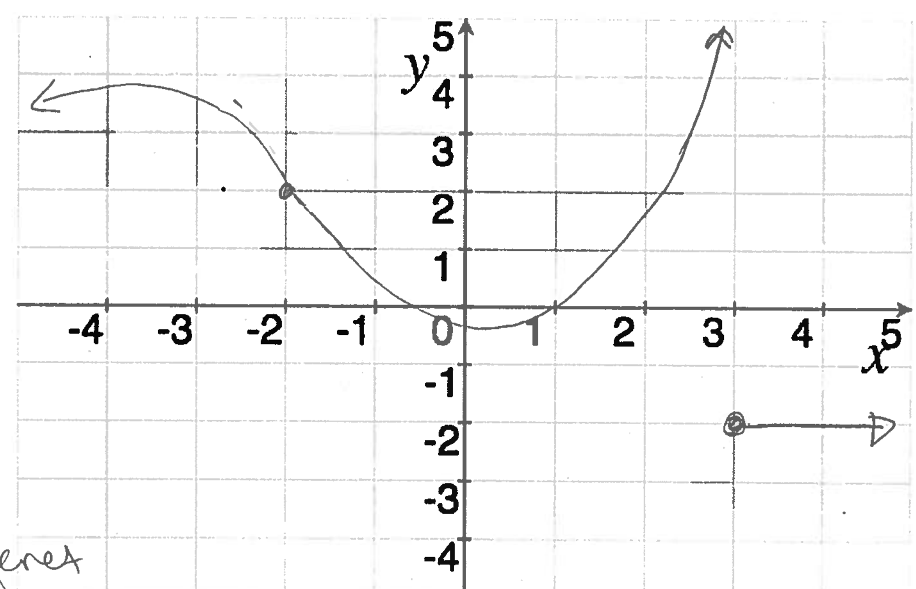
2. [4] (Quiz 2 #3) Sketch the graph of an example function f that satisfies the following conditions:

(a) f is continuous everywhere but when $x = 3$ (+1)

(b) $\lim_{x\to 3^-} f(x) = \infty$ (+1)

(c) $f(-2) = 2$ (+1)

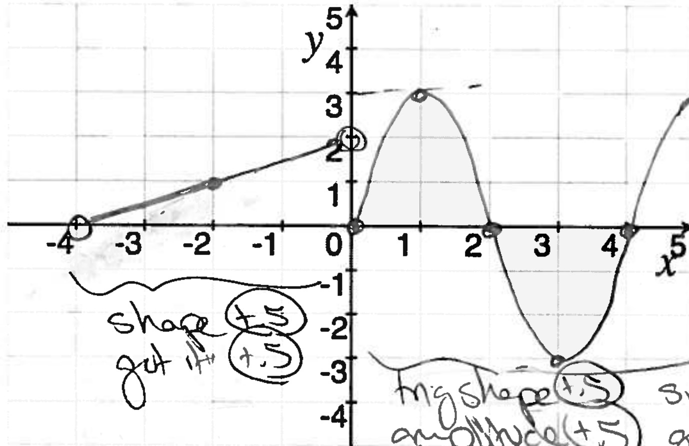
(d) $f'(-2) < 0$ (+1) => slope of line tangent to f when x=-2 is negative



note: one of many correct answers.

3. Let

$$g(x) = \begin{cases} \frac{1}{2}x + 2 & \text{if } -4 < x < 0. \\ 3\sin\left(\frac{\pi}{2}x\right) & \text{if } 0 \le x \end{cases}$$



(a) [4] (Quiz1 #1) *Carefully* graph g below.

endpoints (+1)

shape (+.5)
got it (+.5)

trig shape (+.5)
amplitude (+.5)

sine shape (+.5)
got it (+.5)

$3\sin\left(\frac{\pi}{2}x\right)$
↓ vert stretch by 3
↓ horiz shrink by $\frac{\pi}{2}$

(b) [1] (§2.2 #5) Estimate $\lim_{x \to 0^-} g(x)$

2 (+1)

partial +.5 for 0

(c) [2] (§2.6 #4) Estimate $\lim_{x \to \infty} g(x)$

does not exist. The graph never "settles"

Knows $\lim_{x\to\infty}$ (+1)
got it (+1)

(d) [2] (WebHW5 #6) Estimate $g'(1)$

0 (+1)

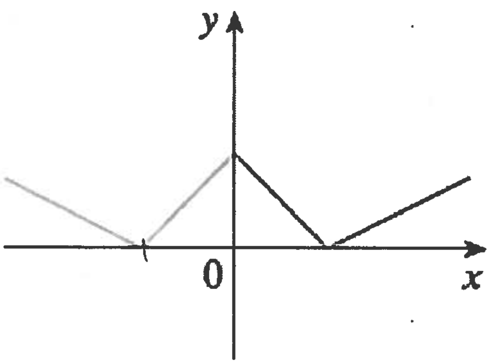
tangent line (+.5)
slope of tang (+.5)

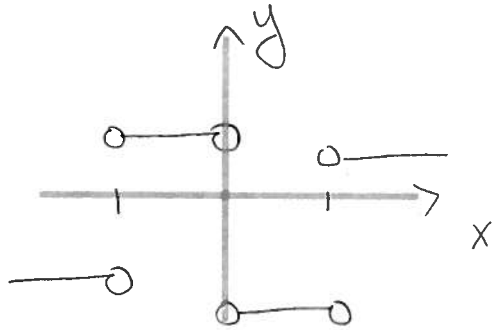
(e) [2] (§2.3 #2) Estimate $\lim_{x \to -2} [6g(x) - 3]$

Limit laws (+1)

$= \lim_{x\to -2} 6g(x) - \lim_{x\to -2} 3$
$= 6 \lim_{x\to -2} g(x) - 3 = 6(1) - 3 = 3$ (+1)

4. [4] (WebHW6 #4) Consider the function m graphed on the left. Sketch m'.





endpoints (+1)
flat (+1)
neg/pos (+2)

5. [12] (§2.3 #3, WebW4 #10, limit laws wks #4, & PracticeExam #4) Find the limit or explain why it does not exist.

notation (+.5)

$\lim_{x\to -1}(2x^2 - x + 1)$ $\boxed{2.5}$

(+1) by continuity of polynomials

$= 2(-1)^2 - (-1) + 1$

(+1) alg $= 2\cdot 1 + 1 + 1$

$= 4$

or

notation (+.5) (+1) by limit laws

$= 2\lim_{x\to -1} x^2 - \lim_{x\to -1} x + \lim_{x\to -1} 1$

alg (+1) $= 2(-1)^2 - (-1) + 1$

$= 2\cdot 1 + 1 + 1 = 4$

$\lim_{h\to 0} \frac{(2+h)^2 - (2+h) - 2}{h}$ $\boxed{2.5}$

limit law (+1) $= \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{2} - h - \cancel{2}}{h}$

algebra (+1) $= \lim_{h\to 0} \frac{3h + h^2}{h}$
notation (+.5)

$= \lim_{h\to 0} \frac{\cancel{h}(3+h)}{\cancel{h}}$

$= \lim_{h\to 0} (3+h)$

$= 3 + 0 = 3$

$\boxed{3}$ $\lim_{x\to\infty} \frac{17 - 7x^2}{8x^2 + 432x}$ $\frac{(1/x^2)}{(1/x^2)}$ notation (+.5)

$= \lim_{x\to\infty} \frac{17/x^2 - 7x^2/x^2}{8x^2/x^2 + 432x/x^2}$

$= \lim_{x\to\infty} \frac{17/x^2 - 7}{8 + \frac{432}{x}}$

(+1) using limit laws:

$= \frac{\lim_{x\to\infty} 17/x^2 - \lim_{x\to\infty} 7}{\lim_{x\to\infty} 8 + \lim_{x\to\infty} 432/x}$

(+.5) b/c 1/Big = Little

(+1) $= \frac{0 - 7}{8 + 0} = -7/8$

$\boxed{4}$ $\lim_{x\to\infty} x^{-6} \sin x$, notation (+.5)

note $-1 \le \sin x \le 1$ (+1)

since $x^{-6} > 0$ for all x
we can multiply the above inequality by x^{-6} to get.

neg exp (+.5) $-x^{-6} \le x^{-6}\sin x \le x^{-6}$ or

alg (+.5) $-\frac{1}{x^6} \le \frac{\sin x}{x^6} \le 1/x^6$

(+1) notice $\lim_{x\to\infty} -1/x^6 = 0 = \lim_{x\to\infty} 1/x^6$

(+.5) by the squeeze theorem then

$\lim_{x\to\infty} x^{-6} \sin x = 0$

partial credit +.5 for zero

6. [5] (poly & exp wks #4) Let $f(x) = x^2 - 3e^x$. Find the equation for the line tangent to the graph of f, when $x = 0$.

Looking for $y = mx + b$ } (.5)

$m =$ slope of line tangent to f when $x = 0$ $= f'(0)$ } (+1)

since $f'(x) = (x^2 - 3e^x)'$
$= (x^2)' - (3e^x)'$
$= 2x - 3(e^x)'$
$= 2x - 3e^x$ } (+1)

we can find
$f'(0) = 2 \cdot 0 - 3 \cdot e^0$
$= -3$ } (+.5)

since the line passes thru $(0, f(0)) = (0, 0^2 - 3e^0)$ or $(0, -3)$ (+1)

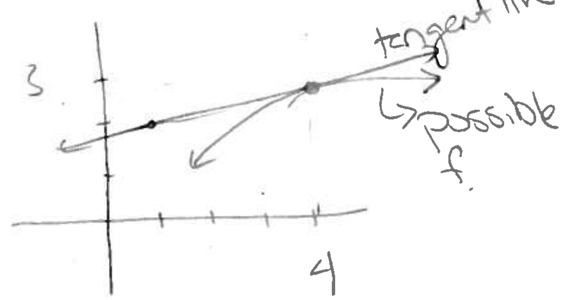
we know
$-3 = (-3)(0) + b$
$\Rightarrow b = -3$

(+.5) Thus $y = -3x - 3$

notation (+.5)

7. [3] (CalcWebHW5 #13) If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$ find the following.

(a) $f(4) = 3$ (+1)



(b) $f'(4)$ = slope of line tangent to $y = f(x)$ when $x = 4$ } (+1)

$= \frac{3-2}{4-0} = \frac{1}{4}$ } (+1)

if confuse f w/ line +1

8. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§2.6 #63) Under certain assumptions the velocity $v(t)$ of a falling raindrop at time t is:

$$v(t) = v^*(1 - e^{\frac{-gt}{v^*}})$$

where g is the acceleration due to gravity (9.8 m/s^2), and v^* is a constant.

i. [3] Find $\lim_{t\to\infty} v(t)$.

ii. [2] Interpret the answer given in (i) as a scientist and explain what v^* is in everyday language.

(b) A rock thrown upwards on planet Mars with velocity $15\frac{\text{m}}{\text{s}}$ has a height $h(t) = 15t - 1.86t^2$ meters t seconds later.

i. [2] (Story wks #6) Find the velocity of the rock after 2 seconds.

ii. [3] (Story wks 6b) Use calculus to find *when* does the rock reach its maximum height?

(a)

i) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty} v^*(1 - e^{-gt/v^*})$

$= v^* \lim_{t\to\infty}(1 - e^{-gt/v^*})$

$= v^*\left[\lim_{t\to\infty} 1 - \lim_{t\to\infty} e^{-gt/v^*}\right]$

$= v^*\left[1 - \lim_{t\to\infty} \frac{1}{e^{gt/v^*}}\right]$

b/c $\frac{1}{\text{Big}}$ = Little

$= v^*[1 - 0]$

$= v^*$

notation (+.5)
limit law (+1)
neg exp (+.5)
Big-Little (+1.5)
alg (+.5)

ii) If the raindrop is allowed to fall the velocity will approach the value v^*. We tend to call v^* the (+1) terminal velocity of the raindrop in everyday language

sense (+1)

b) $h(t) = 15t - 1.96t^2$

i) the velocity function $v = h'(t)$ (+.5)

(+1)

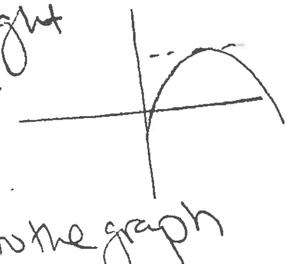
$$
\begin{aligned}
\text{note } h'(t) &= (15t - 1.96t^2)' \\
&= (15t)' - (1.96t^2)' \\
&= 15 - 1.96(t^2)' \\
&= 15 - 1.96 \cdot 2t \\
&= 15 - 3.72t
\end{aligned}
$$

So

(+.5)

$$
\begin{aligned}
v(2) = h'(2) &= 15 - 3.72(2) \\
&= 15 - 7.44 \\
&= 7.56
\end{aligned}
$$

ii) the rock hits the max height when the slope of the line tangent to the graph of h is zero. (+1)



So when $h'(x) = 0$ (+.5) (+.5)

$\Rightarrow 15 - 3.72t = 0$

alg (+1) $\Rightarrow -15 = -3.72t$

$\rightarrow t = \frac{15}{3.72}$

$\approx 4.03s$

+1 if use precalc/tables