

3. Find:

$$\frac{d}{dx} \left(\frac{e^x}{3x+2} \right)$$

$$= \frac{(3x+2) \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(3x+2)}{[3x+2]^2}$$

$$= \frac{(3x+2)e^x - e^x(3)}{[3x+2]^2}$$

$$= \frac{e^x[3x+2-3]}{[3x+2]^2} = \frac{e^x(3x-1)}{(3x+2)^2}$$

$$\left(\frac{3x^2 - \sqrt{x}}{x} \right)'$$

$$= \frac{x(3x^2 - x^{\frac{1}{2}})' - (3x^2 - x^{\frac{1}{2}})(x)'}{(x)^2}$$

product rule:

$$= \frac{x(6x - \frac{1}{2}x^{-\frac{1}{2}}) - (3x^2 - x^{\frac{1}{2}})(1)}{(3x^2 - x^{\frac{1}{2}})(x^{-1})' + x^{-1}(3x^2 - x^{\frac{1}{2}})'} = \frac{6x^2 - \frac{1}{2}x^{\frac{1}{2}} - 3x^2 + x^{\frac{1}{2}}}{(3x - x^{\frac{1}{2}})(-x^{-2}) + x^{-1}(6x - \frac{1}{2}x^{-\frac{1}{2}})}$$

$$= \frac{3x^2 + \frac{1}{2}x^{\frac{1}{2}}}{-x^{-2} + \frac{6x - \frac{1}{2}x^{-\frac{1}{2}}}{x}} = 3 + \frac{1}{2x^{\frac{3}{2}}}$$

4. Consider the function $f(x) = \frac{6x}{1+3x^2}$.

(a) Find the equation of the line tangent to f when $x = 3$

Looking for $y = mx + b$
 $m =$ slope of line tangent to f when $x = 3$

from *

$$= f'(3)$$

$$= \frac{(1+3(3)^2)6 - (6(3))(6(3))}{(1+3(3)^2)^2}$$

$$= \frac{28 \cdot 6 - 18 \cdot 18}{28^2} \approx -0.199$$

finding $f'(x)$:

$$f'(x) = \frac{(1+3x^2)(6x)' - (6x)(1+3x^2)'}{(1+3x^2)^2}$$

$$= \frac{(1+3x^2)6 - 6x(6x)}{(1+3x^2)^2}$$

passes thru $(3, f(3)) = (3, \frac{6(3)}{1+3(3)^2}) \approx (3, 0.6)$
 So $0.6(3) = (-0.199)(3) + b$
 $\Rightarrow b \approx 1.24$ so $y = -0.199x + 1.24$

(b) Find where the function f has a horizontal tangent line.

Find x so that

slope of line tangent to f at x = slope of horizontal line

$$\Rightarrow f'(x) = 0 \quad (\text{from } *)$$

$$\Rightarrow \frac{(1+3x^2)6 - 6x(6x)}{(1+3x^2)^2} = 0$$

$$\Rightarrow \frac{6 + 18x^2 - 36x^2}{(1+3x^2)^2} = 0$$

$$\frac{6 - 18x^2}{(1+3x^2)^2} = 0$$

$$\Rightarrow 6 - 18x^2 = 0$$

$$\Rightarrow 6 = 18x^2$$

$$\Rightarrow \frac{1}{3} = x^2$$

$$\Rightarrow x = \pm \sqrt{\frac{1}{3}}$$