

# TMATH 124am: Quiz 3

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [5] (product & quotient wks #4) Let  $f(x) = \frac{6x}{1+3x^2}$

notation (+5)

(a) Find the slope of the line tangent to the graph of  $f$  when  $x = 2$ .

Slope of line tangent to  $f$  when  $x = 2$  =  $f'(2)$  (+5)

finding  $f'(x)$ :

$$f'(2) = \frac{(1+3(2)^2)(6) - (6(2))(6(2))}{(1+3(2)^2)^2}$$

$$f'(x) = \frac{(1+3x^2)(6x)' - 6x(1+3x^2)'}{(1+3x^2)^2}$$

$$= \frac{(1+3 \cdot 4) \cdot 6 - 12 \cdot 12}{(1+3 \cdot 4)^2}$$

$$= \frac{(1+3x^2)(6) - 6x(6x)}{(1+3x^2)^2}$$

$$= \frac{13 \cdot 6 - 12 \cdot 12}{(1+3 \cdot 4)^2}$$

$$= \frac{78 - 144}{169} = \frac{-66}{169} \approx .39$$

$$\frac{13}{73} \quad \frac{13}{9}$$

(b) Find where the function  $f$  has a horizontal tangent line.

find  $x$  so that

slope of line tangent to  $f$  at  $x$  = slope of a horizontal line

$$\Rightarrow (+5) \quad f'(x) = 0$$

from part (a)

$$(+5) \quad \frac{(1+3x^2)(6) - 6x(6x)}{(1+3x^2)^2} = 0$$

$$\frac{6}{13} = \frac{18x^2}{13}$$

$$\Rightarrow (1+3x^2)6 - 6x(6x) = 0$$

$$\frac{1}{3} = x^2$$

$$\Rightarrow 6 + 18x^2 - 36x^2 = 0$$

$$\pm \sqrt{\frac{1}{3}} = x$$

$$\Rightarrow 6 - 18x^2 = 0$$

$$\approx \pm .58$$

Algebra (+1)

2. [2] (§3.3 #43) Find  $\lim_{x \rightarrow 0} \frac{5x^3 - 4x}{\sin(3x)} = \lim_{x \rightarrow 0} \frac{x(5x^2 - 4)}{\sin 3x}$

use  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

$= \lim_{x \rightarrow 0} \frac{3 \cdot x}{3 \cdot \sin 3x} \cdot \frac{(5x^2 - 4)}{1}$

$= \lim_{x \rightarrow 0} \frac{3x}{\sin 3x} \cdot \frac{5x^2 - 4}{3}$

$= \lim_{x \rightarrow 0} \frac{3x}{\sin 3x} \cdot \lim_{x \rightarrow 0} \frac{5x^2 - 4}{3} = 1 \cdot \frac{5(0)^2 - 4}{3} = \frac{-4}{3}$

algebra (+.5)  
limit laws (+.5)

3. [3] (WebHW9 #8) Let  $g(x) = 5^{x^3 \sin(x)}$ . Find  $g'(x)$  and simplify.

$5^{x^3 \sin(x)}$

(+.5) Chain Rule

$f(u) = 5^u$

$f'(u) = 5^u \ln 5$  (+.5)

$g(x) = x^3 \sin(x)$

$g'(x) = x^3 (\sin(x))' + \sin(x) (x^3)' = x^3 \cos(x) + (\sin(x)) 3x^2$

note  $f(g(x)) = f(x^3 \sin(x)) = 5^{x^3 \sin(x)}$  ✓

$[5^{x^3 \sin(x)}]' = f'(g(x)) g'(x)$

$= f'(x^3 \sin(x)) \cdot [x^3 \cos(x) + (\sin(x)) 3x^2]$

$= 5^{x^3 \sin(x)} \cdot \ln 5 \cdot [x^3 \cos x + 3x^2 \sin x]$  } used chain rule (+.5)