Key

TMATH 124am: Quiz 2

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. (§2.7 #4) Let $f(x) = x - x^3$

   (a) [2] Find the slope of the line tangent to the graph of f when $x = 2$.

slope of line tangent to the graph of f when x = 2 $= f'(2)$ (+.5)

$$= \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2} \frac{(x-x^3)-(2-2^3)}{x-2} \quad (+.5)$$

$$= \lim_{x\to 2} \frac{x-x^3+6}{x-2}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(-x^2-2x-3)}{\cancel{x-2}}$$

$$= \lim_{x\to 2} (-x^2-2x-3)$$

$$= -(2)^2-2(2)-3 = -11$$

$$\begin{array}{r} -x^2-2x-3 \\ x-2 \overline{\big) -x^3+0x^2+x+6} \\ (-x^3+2x^2) \\ \hline -2x^2+x+6 \\ -(-2x^2+4x) \\ \hline -3x+6 \\ -3x+6 \end{array}$$

or $= f'(2)$ (+.5)

$$= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0} \frac{[(2+h)-(2+h)^3]-[2-2^3]}{h}$$

$$= \lim_{h\to 0} \frac{2+h-(8+3\cdot 4\cdot h+3\cdot 2h^2+h^3)+6}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2}+h-\cancel{8}-12h-6h^2-h^3+\cancel{6}}{h} \quad \text{alg } (+.5)$$

$$= \lim_{h\to 0} \frac{-11h-6h^2-h^3}{h} = \lim_{h\to 0} \frac{\cancel{h}(-11-6h-h^2)}{\cancel{h}}$$

$$= \lim_{h\to 0} -11-6h-h^2 = -11-6\cdot 0-0^2 = -11$$

limit law (+.5)

   (b) [2] Find the equation of the line tangent to the graph of f when $x = 2$.

Looking for $y = mx+b$ (+.5)

m = slope of line tangent to the graph of f when x = 2 $= f'(2) = -11$ from part (a) (+.5)

passes thru the point $(2, f(2)) = (2,-6)$ (+.5)

So $-6 = -11(2)+b$

$\Rightarrow -6+22 = b$

$16 = b$ (+.5)

So $y = -11x+16$

2. [2] (WebHW4 #11) Let $g(x) = \dfrac{x-8}{x^2+5}$. Find $\lim_{x\to-\infty} g(x)$.

$\lim_{x\to-\infty} \dfrac{x-8}{x^2+5} \dfrac{(1/x^2)}{(1/x^2)}$ +.5

$= \lim_{x\to-\infty} \dfrac{x/x^2 - 8/x^2}{x^2/x^2 + 5/x^2}$

$= \lim_{x\to-\infty} \dfrac{1/x - 1/x^2}{1 + 5/x^2} \to 0 = \dfrac{0}{1} = 0$

algebra +.5 or

Big-Little +.5

+.5

denominator is growing faster than numerator

$\dfrac{1}{\text{Big}}$ = little so 0

3. [4] (Con't Wks #6) Sketch a graph of a function α that satisfies *all* of the following:

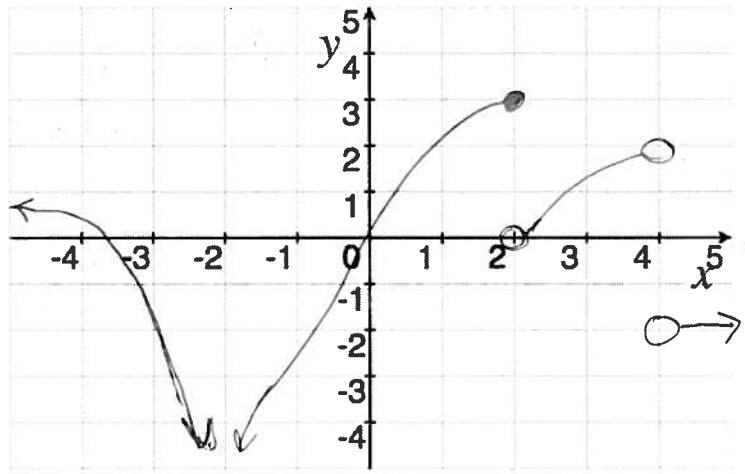
$\lim_{x\to2^+} \alpha(x) = 0$ +.5

$\lim_{x\to-2} \alpha(x) = -\infty$ +1

$\alpha(2) = 3$ +.5

α is discontinuous when $x = 4$ +1

$\lim_{x\to-\infty} \alpha(x) = 1$ +1



one of many correct answers