3. For each f defined below, find $f'(x)$.

$f(x) = x^4 + 2e^x$ $\qquad$ $f(x) = e^{x+4} - 7e^2$ $\qquad$ $f(x) = \dfrac{e^x + 7}{e}$

$f'(x) = [x^4 + 2e^x]'$
$= [x^4]' + [2e^x]'$
$= 4x^3 + 2[e^x]'$
$= 4x^3 + 2e^x$

$\frac{d}{dx}f(x) = \frac{d}{dx}[e^{x+4} - 7e^2]$
$= \frac{d}{dx}[e^4e^x - 7e^2]$
$= \frac{d}{dx}[e^4e^x] - \frac{d}{dx}[7e^2]$
$= \underset{\text{constant}}{e^4}\frac{d}{dx}[e^x] - \frac{d}{dx}[\underset{\text{constant}}{7e^2}x^0]$
$= e^4e^x - 0 = e^{4+x}$

$f'(x) = \left[\frac{e^x}{e} + \frac{7}{e}\right]'$
$= [\frac{1}{e}e^x + \frac{7}{e}]'$
$= [\frac{1}{e}e^x]' + [\frac{7}{e}x^0]'$
$= \underset{\text{constant}}{\frac{1}{e}}[e^x]' + \underset{\text{constant}}{\frac{7}{e}}[x^0]'$
$= \frac{1}{e}e^x + 0 = e^{x-1}$

4. Consider $\alpha(x) = x^4 + 2e^x$.

(a) Find the equation of the line tangent to the graph of α at the point $(0, 2)$.

Looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$

m = slope of line tangent to α when $x=0$ $= \alpha'(0)$ from #3 we know $\alpha'(x) = 4x^3 + 2e^x$

so $\alpha'(0) = 4 \cdot 0^3 + 2 \cdot e^0 = 0 + 2 \cdot 1 = 2$

$\Rightarrow m = 2$. The line passes thru $(0,2)$ so $2 = 2 \cdot (0) + b \Rightarrow b = 2$

$\Rightarrow y = 2x + 2$ or $y - 2 = 2(x - 0)$

(b) Find the line normal to the line you found in part (a) that also passes through the point $(0, 2)$.

Recall two lines are normal if their slopes are perpendicular. The line in (a) has slope 2 so a perpendicular line has slope $-\frac{1}{2}$. The line passes thru $(0,2)$ so $2 = -\frac{1}{2}(0) + b \Rightarrow b = 2$ $\Rightarrow y = -\frac{1}{2}x + 2$

5. At what point on the curve of $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?

Let $\beta(x) = 1 + 2e^x - 3x$. We want to find x so that

the slope of the line tangent to β at x = slope of $3x - y = 5$

notice $3x - y = 5$
$3x - 5 = y$

— or —

$\beta'(x) = 3$

we can compute $\beta'(x) = [1 + 2e^x - 3x]'$
$= 2e^x - 3$

$2e^x - 3 = 3$
$2e^x = 6$

$e^x = 6/2$
$e^x = 3$

$\ln e^x = \ln 3$
$x = \ln 3$