Implicit Differentiation Practice

1. Assume that y is a function of x. Find $\frac{dy}{dx}$ in the following:

   (a) $x^3 + y^3 = 8$

   (b) $y\cos(x) = x^2 + y^2$

   (c) $e^{xy} = e^{3x} - e^{4y}$

$$\underbrace{\frac{d}{dx}[e^{xy}]}_{\text{by } *} = \underbrace{\frac{d}{dx}[e^{3x}]}_{\text{by } \dagger} - \underbrace{\frac{d}{dx}[e^{4y}]}_{\text{by } ☺}$$

$$e^{xy}\left(x\frac{dy}{dx} + y\right) = e^{3x}\cdot 3 - e^{4y}\cdot 4\frac{dy}{dx}$$

$$e^{xy}x\frac{dy}{dx} + e^{xy}y = e^{3x}\cdot 3 - e^{4y}4\frac{dy}{dx}$$

$$e^{xy}x\frac{dy}{dx} + e^{4y}4\frac{dy}{dx} = e^{3x}\cdot 3 - e^{xy}y$$

$$\frac{dy}{dx} = \frac{3e^{3x} - ye^{xy}}{e^{xy}x + e^{4y}4}$$

$*$ $\frac{d}{dx}[e^{xy}]$

$f(u) = e^u \quad f'(u) = e^u$

$g(x) = x\cdot y \quad g'(x) = x[y]' + y[x]' = x\frac{dy}{dx} + y(1)$

$\Rightarrow \frac{d}{dx}[e^{xy}] = f'(g(x))\,g'(x) = f'(xy)\cdot\left[x\frac{dy}{dx} + y\right] = e^{xy}\left(x\frac{dy}{dx} + y\right)$

$\dagger$ $\frac{d}{dx}[e^{3x}]$

$f(u) = e^u \quad f'(u) = e^u$

$g(x) = 3x \quad g'(x) = 3$

$\frac{d}{dx}[e^{3x}] = f'(g(x))\,g'(x) = e^{3x}\cdot 3$

☺ $\frac{d}{dx}[e^{4y}]$

$f(u) = e^u \quad f'(u) = e^u$

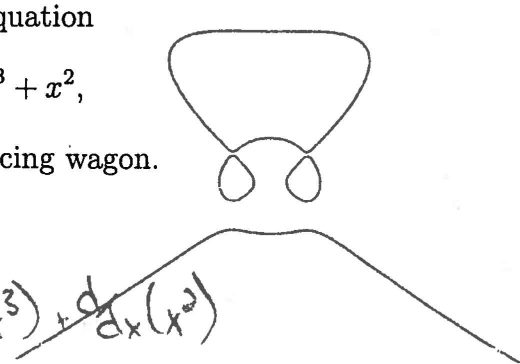
$g(x) = 4y \quad g'(x) = 4\frac{dy}{dx}$

$\frac{d}{dx}(e^{4y}) = f'(g(x))\,g'(x) = e^{4y}\cdot 4\frac{dy}{dx}$

2. It is mentioned in §3.5 #42 that the graph of the equation

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2,$$

as seen to the right, without axes, looks like a bouncing wagon. Find $\frac{dy}{dx}$.



$$\frac{d}{dx}(2y^3 + y^2 - y^5) = \frac{d}{dx}(x^4 - 2x^3 + x^2)$$

$$\frac{d}{dx}(2y^3) + \frac{d}{dx}(y^2) - \frac{d}{dx}(y^5) = \frac{d}{dx}(x^4) - \frac{d}{dx}(2x^3) + \frac{d}{dx}(x^2)$$

$$2\cdot 3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5y^4\frac{dy}{dx} = 4x^3 - 2\cdot 3x^2 + 2x$$

$$6y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5y^4\frac{dy}{dx} = 4x^3 - 6x^2 + 2x$$

$$\frac{dy}{dx}(6y^2 + 2y - 5y^4) = 4x^3 - 6x^2 + 2x \Rightarrow \frac{dy}{dx} = \frac{4x^3 - 6x^2 + 2x}{6y^2 + 2y - 5y^4}$$