TMATH 124pm: Quiz 5

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. There are two sides of this quiz.

1. [2] (WebHW13 #12) If $g(2) = 7$ and $g'(x) \leq 1$ for $2 \leq x \leq 5$, how small can $g(5)$ possibly be? Briefly justify your answer.

Since $g'(x) \leq 1$ for x between 2 & 5 we know g is differentiable on $(2,5)$. Since g is differentiable on $(2,5)$, we know g is con't on $(2,5)$ by results from §3.1.

Thus we can use the MVT.

So there exists a c between 2 and 5 so that

$$g'(c) = \frac{g(5) - g(2)}{5-2}$$

$$\Rightarrow g'(c) = \frac{g(5) - 7}{3}$$

Since $g'(c) \leq 1$ the above implies

$$\frac{g(5) - 7}{3} \leq 1$$

$$\Rightarrow g(5) - 7 \leq 3$$

$$\Rightarrow g(5) \leq 10$$

so $g(5)$ can be as large as 10

So we don't know how small g can be.

2. [6] Let $f(x) = \frac{(\ln x)^2}{x}$. For parts (b) and (c), restrict to the interval $(0, 20)$.

(a) [2] (§4.4 #30) Find $\lim_{x\to\infty} f(x)$.

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x} \underset{L'H}{=} \lim_{x\to\infty} \frac{2\ln x \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{2\ln x}{x} \quad \text{"}\frac{\infty}{\infty}\text{"}$$

"$\frac{\infty}{\infty}$" (+.5) applying L'H

(+.5) (+.5)

$$\underset{L'H}{=} \lim_{x\to\infty} \frac{2\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{2}{x} = 0$$

do again (+.5)

(b) [5] (wks #2) Find the x coordinate of any local extrema on the interval $[0, 20]$.

Find CP: } looking for (+.5)

$$f'(x) = [(\ln x)^2 x^{-1}]'$$
$$= 2\ln x \frac{1}{x} x^{-1} + (\ln x)^2 x^{-2}(-1)$$
$$= \frac{2\ln x}{x^2} - \frac{(\ln x)^2}{x^2} = \frac{2\ln x - (\ln x)^2}{x^2}$$
$$= \frac{\ln x(2-\ln x)}{x^2} \Big\} \text{(+1.5)}$$

domain of f': $x \neq 0$

$$0 = \frac{(\ln x)(2-\ln x)}{x^2} \Big\} \text{(+.5)}$$

$0 = (\ln x)(2-\ln x)$ alg (+1) plen (+.5)

$0 = \ln x$ or $0 = 2 - \ln x$

$x = 1$ or $x = e^2$ both are extrema

$x = 20$ could count too..

$0 \quad 1 \quad e^2$

analysis of extrema (+1)

$\frac{\ln e^{-1}(2-\ln e^{-1})}{(e^{-1})^2}$ $\frac{-1 \cdot 2}{+}$ neg

$\frac{\ln e(2-\ln e)}{e^2}$ $\frac{+\ +}{+}$ pos

$\frac{\ln e^3(2-\ln e^3)}{(e^3)^2}$ $\frac{+\ -}{+}$ neg

(c) [1] Find the value (y coordinate) of the absolute minimum on the interval $(0, 20)$.

given our work in (b)

the abs min would happen at either 1 or 20 } (+.5)

$$f(1) = \frac{(\ln 1)^2}{1} = 0 \quad \text{or} \quad f(20) = \frac{(\ln 20)^2}{20} > 0 \Big\} \text{compare (+.5)}$$

so 0 is the abs min.