Would you like to take a quiz 3/7 over sections 4.1, 4.2 & 4.4 where you can use your 3x5" double-sided notecard?

Yes (take) No (don't)

TMATH 124pm: Quiz 4   Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. There are two sides of this quiz.

1. [~~X~~ 5] Find $\frac{dy}{dx}$ for the following:

[2] (WebHW10 #6)
$y = \underbrace{2x\log(x)}_{\text{product (+1)}}$

$y' = 2x[\log(x)]' + [2x]'\log(x)$

$= \underbrace{2x\frac{1}{x\ln 10}}_{(+.5)} + 2\cdot\log(x)$

$= \frac{2}{\ln 10} + 2\log(x)$

notation/alg (+.5)

[3] (LogDif Wks #2)
$y = x^{\sqrt{x}}$

into ln (+.5)
prop ln (+.5)

$\ln y = \ln x^{\sqrt{x}}$

$\ln y = \sqrt{x}\ln x$

$\ln y = \underbrace{x^{1/2}\ln x}_{\text{product (+1)}}$

$\frac{d}{dx}(\ln y) = x^{1/2}[\ln x]' + [x^{1/2}]'\ln x$

$\underbrace{\frac{1}{y}\cdot\frac{dy}{dx}}_{(+.5)} = x^{1/2}\cdot\frac{1}{x} + \frac{1}{2}x^{-1/2}\ln x$

$\frac{dy}{dx} = y\left[\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right]$

or

$x^{\sqrt{x}}\left[\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right]$

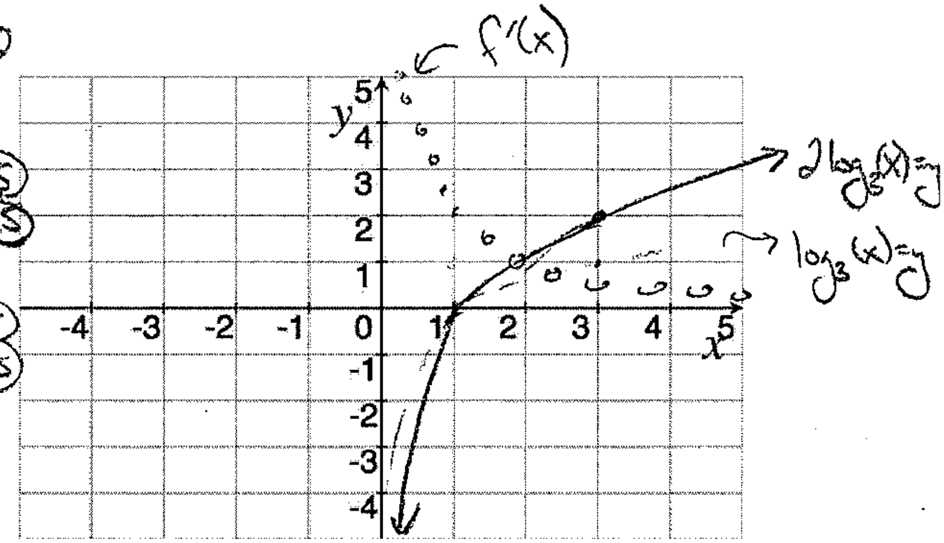
got it (+.5)

vert stretch by 2

2. Let $f(x) = 2\log_3(x)$.

(a) [1] Graph $f(x)$ on the axis provided shape (.5) transform (.5)

(b) [1] Sketch the graph of $f'(x)$ on the axis provided. shape (.5) positive (.5)



$$f'(x) = 2[\log_3(x)]'$$
$$= 2\frac{1}{x(\ln 3)} = \frac{2}{x(\ln 3)}$$

(c) [3] (§3.6 #33) Find the equation of the line that is tangent to f when $x = 3$.

looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$ (.5)

$m =$ slope of line tangent to f when $x = 3$ $= f'(3)$ (.5)

note $f'(x) = \frac{2}{x(\ln 3)}$ (.5) so $f'(3) = \frac{2}{3(\ln 3)}$ (.5)

The line touches the curve f at $(3, f(3)) = (3, 2\log_3 3)$ (.5)
or $(3, 2\cdot 1) = (3, 2)$

plug in (.5)

So

$2 = \frac{2}{3(\ln 3)}(3) + b$ or $y - 2 = \frac{2}{3(\ln 3)}(x - 3)$

$2 - \frac{2}{\ln 3} = b$

$\Rightarrow y = \frac{2}{3(\ln 3)}x + \left(2 - \frac{2}{\ln 3}\right)$

$\approx .606x + .18$

Do you want to take a quiz on 3/7 over sections 4.1, 4.2 & 4.4 where you can use your 3x5" double-sided notecard?

Yes (take) 8 No (don't) 9

Key

TMATH 124am: Quiz 4

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. There are two sides of this quiz.

1. Find $\frac{dy}{dx}$ for the following:

[2] (WebHW10 #3)
$y = \log(x^6 + 5)$

$$\frac{d}{dx}(y) = \frac{d}{dx}\underbrace{(\log(x^6+5))}_{\text{chain } (*)\ (+1)}$$

$$\frac{dy}{dx} = \frac{6x^5}{(x^6+5)(\ln 10)}$$

(*) $g(x) = x^6 + 5$ $g'(x) = 6x^5$

$f(u) = \log u$ (+.5) $f'(u) = \frac{1}{u(\ln 10)}$

$f(g(x)) = \log(x^6+5)$ ✓

$f'(g(x))\,g'(x) = f'(x^6+5) \cdot 6x^5$

$= \frac{1}{(x^6+5)(\ln 10)} \cdot 6x^5$

alg/notation (+.5)

[3] (LogDif Wks #1b)
$y = (\sin x)^{\ln x}$

intro ln (+.5)
prop ln (+.5)

$\ln y = \ln[(\sin x)^{\ln x}]$

$\ln y = \underbrace{(\ln x)\ln(\sin x)}_{\text{product rule } (+.5)}$

$\frac{d}{dx}$

$\underbrace{\frac{1}{y}\frac{dy}{dx}}_{(+.5)} = \ln x\,\underbrace{[\ln(\sin x)]'}_{\text{chain } (+)\ (+.5)} + [\ln x]'\ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = (\ln x)\frac{\cos x}{\sin x} + \frac{\ln(\sin x)}{x}$

$\frac{dy}{dx} = y\left[(\ln x)\frac{\cos x}{\sin x} + \frac{\ln(\sin x)}{x}\right]$

or

$= (\sin x)^{\ln x}\left[\ln x\frac{\cos x}{\sin x} + \frac{\ln(\sin x)}{x}\right]$

got + (+.5)

(+) $g(x) = \sin(x)$ $g'(x) = \cos x$

$f(u) = \ln u$ $f'(u) = 1/u$

$f(g(x)) = \ln(\sin(x))$ ✓

$f'(g(x))\,g'(x) = f'(\sin x)\cdot\cos x$

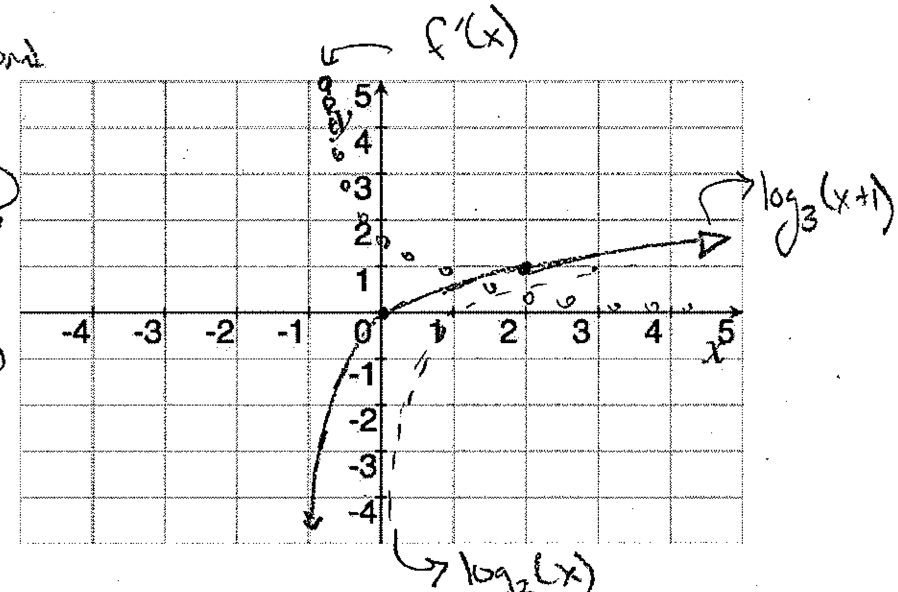
$= \frac{1}{\sin x}\cdot\cos x$

shift left 1 unit

2. Let $f(x) = \log_3(x+1)$.

(a) [1] Graph $f(x)$ on the axis provided shape .5 transform .5

(b) [1] Sketch the graph of $f'(x)$ on the axis provided. shape .5 positive .5



$$f'(x) = \underbrace{[\log_3(x+1)]'}_{\text{chain (!!)}}$$

$$= \frac{1}{(x+1)(\ln 3)} \cdot (1)$$

(c) [3] (§3.6 #33) Find the equation of the line that is tangent to f when $x = 2$.

looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$ .5

m = slope of line tangent to f when $x = 2$ $= f'(2)$. .5

note $f'(x) = \dfrac{1}{(x+1)\ln 3}$ .5 so $f'(2) = \dfrac{1}{(2+1)\ln 3} = \dfrac{1}{3\ln 3} \approx .303$ .5

The line touches the curve f at $(2, f(2)) = (2, \log_3(2+1))$ .5

or $(2, 1)$ plug in .5

So

$$1 = \frac{1}{3\ln 3} \cdot 2 + b$$

or

$$y - 1 = \frac{1}{3\ln 3}(x - 2)$$

$$\Rightarrow b = 1 - \frac{2}{3\ln 3}$$

$$\Rightarrow y = \frac{1}{3\ln 3} x + \left(1 - \frac{2}{3\ln 3}\right)$$

(!!) chain

$g(x) = x + 1$ $g'(x) = 1$

$f(u) = \log_3 u$ $f'(u) = \frac{1}{u(\ln 3)}$

$f(g(x)) = \log_3(x+1)$ ✓

$f'(g(x))g'(x) = \frac{1}{(x+1)(\ln 3)}$