Key

TMATH 124pm: Quiz 3

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. There are two sides of this quiz.

1. [2] (§3.3 #41) Find the *limit*: $\lim_{x\to 0} \dfrac{\sin(6x)}{\cos(6x)\sin(2x)}$

$$\lim_{x\to 0} \frac{\sin 6x}{6x} \cdot \frac{6x}{\cos(6x)\sin(2x)}$$

$$= \lim_{x\to 0} \frac{\sin 6x}{6x} \cdot \lim_{x\to 0} \frac{3}{\cos(6x)} \cdot \frac{2x}{\sin(2x)}$$

$$= 1$$

$$= \lim_{x\to 0} \frac{3}{\cos(6x)} \cdot \lim_{x\to 0} \frac{2x}{\sin(2x)}$$

$$= \frac{3}{\cos(6\cdot 0)} \cdot \lim_{x\to 0} \frac{1}{\frac{\sin(2x)}{2x}}$$

$$= \frac{3}{\cos(0)} = 3$$

notation (+.5)

alg (+.5)

seq (+.5) $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

multiplies (+.5)

2. (Exp Wks #3) Let $f(x) = 1 + 2e^x - 3x$.

(a) [1] Find $\frac{d}{dx} f(x)$.

$\frac{d}{dx}(1+2e^x-3x) = \frac{d}{dx}(1) + \frac{d}{dx}(2e^x) - \frac{d}{dx}(3x)$

$= 0 + 2\frac{d}{dx}(e^x) - 3\frac{d}{dx}(x) = \underbrace{2e^x}_{+.5} - \underbrace{3}_{+.5}$

(b) [3] At what x value is the line tangent to f also parallel to the line $3x - y = 5$?

note the slope of $3x - y = 5$ or $3x - 5 = y$ is 3.

We want to find when the slope of the line tangent to f is 3 (+1) — find x so that $f'(x) = 3$

$\Rightarrow 2e^x - 3 = 3$ (+.5 alg) (use ln +.5 property ln +.5)

$\Rightarrow 2e^x = 6$

$\Rightarrow e^x = 3$ (+.5)

$\Rightarrow x = \ln(3)$

3. (WebHW7 #8) Let $g(x) = x + \cos(x)$.

(a) [1] Find $g'(x)$.

$g'(x) = [x + \cos(x)]' = [x]' + [\cos(x)]' = \underbrace{1}_{+.5} - \underbrace{\sin(x)}_{+.5}$

(b) [3] Find the equation of the line that is tangent to g when $x = \frac{-\pi}{6}$.

Looking for $y = mx + b$ (+.5)

m = slope of the line tangent to g when $x = -\pi/6$ $= g'(-\pi/6)$ (+1)

$= 1 - \sin(-\pi/6)$

$= 1 - (-\frac{1}{2}) = 1 + \frac{1}{2} = \frac{3}{2}$ (+.5)

The line is tangent to g when $x = -\pi/6$ so the line shares the point (+.5)

$(-\pi/6, g(-\pi/6)) = (-\pi/6, -\pi/6 + \cos(-\pi/6))$ with the graph of g.

$\Rightarrow$ if $x = -\pi/6$ $y = -\pi/6 + \cos(-\pi/6) = -\pi/6 + \frac{\sqrt{3}}{2}$ (+.5)

$\Rightarrow -\pi/6 + \frac{\sqrt{3}}{2} = (\frac{3}{2})(-\pi/6) + b$ (+.5)

$\Rightarrow b = -\pi/6 + \sqrt{3}/2 + \frac{3\pi}{2 \cdot 6} = -\pi/6 + \pi/4 + \sqrt{3}/2 = \pi/12 + \sqrt{3}/2$

So $y = \frac{3}{2}x + (\pi/3 + \sqrt{3}/2)$