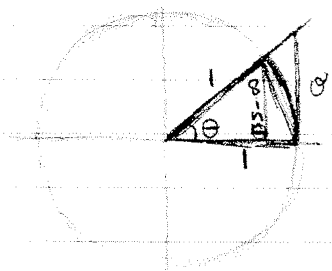
Exam 1 Extra Credit [redacted]

10) use squeeze thrm to prove $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

to apply the squeeze thrm, you need a function smaller than $\frac{\sin\theta}{\theta}$ and a function greater than $\frac{\sin\theta}{\theta}$.

to derive the two necessary functions, you use visual trig w/ the unit circle.



assume that θ is between 0 and $\frac{\pi}{2}$
now let's draw colorful Δ's

by def of the unit circle, the radius is 1.

also by def, $\sin\theta$ is the y-coordinate of the unit circle (the height of the red Δ)

recall SOHCAHTOA, tangent is $\frac{\text{opposite}}{\text{adjacent}}$.

$\tan\theta = \frac{\sigma}{1}$
$= \sigma$ (which is the height of blue Δ)

w/ the above info, visually derive: $\Delta < \Delta < \Delta$ and compute each area.

area of Δ: $A = \frac{1}{2}bh$. for red $\Delta = \frac{1}{2} \cdot 1 \cdot \sin\theta$
$= \frac{1}{2}\sin\theta$

for blue $\Delta = \frac{1}{2} \cdot 1 \cdot \tan\theta$
$= \frac{1}{2}\tan\theta$

area of sector: $A = \frac{\theta}{2\pi} \cdot \pi r^2$ (area of circle)

for green $\Delta = \frac{\theta}{2\pi} \cdot \pi (1)^2$
$= \frac{\theta}{2\pi} \cdot \pi = \frac{\theta}{2}$

so the area of $\triangle$ < area of ⌔ < area of $\triangle$

$\rightarrow \frac{1}{2}\sin\theta < \frac{\theta}{2} < \frac{1}{2}\tan\theta$

$\times 2$ across: $\sin\theta < \theta < \tan\theta$

but note: this is true in quadrant 1 (where all values are positive)

recall that we're trying to prove $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

so we need both positive and negative values of θ to prove the limit.

($[-\frac{\pi}{2}, \frac{\pi}{2}]$ aka quad 1 and quad 4)

so you take the abs value of all sides of inequality.

$|\sin\theta| < |\theta| < |\tan\theta|$ (the justification that this holds when $\theta < 0$ is the same as above but mirrored below the x axis)

$\div |\sin\theta|$ across: $\frac{|\sin\theta|}{|\sin\theta|} < \frac{|\theta|}{|\sin\theta|} < \frac{|\tan\theta|}{|\sin\theta|}$

$\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$ $= 1 < \frac{|\theta|}{|\sin\theta|} < \frac{\left|\frac{\sin\theta}{\cos\theta}\right|}{|\sin\theta|} = 1 < \frac{|\theta|}{|\sin\theta|} < \frac{1}{|\cos\theta|}$

invert the inequality: (ex if $\frac{1}{2} < \frac{1}{3} \Rightarrow 2 > 3$)

$$1 > \frac{|\sin\theta|}{|\theta|} > |\cos\theta|$$

(in Q1: $\frac{\text{positive}}{\text{positive}}$, in Q4: $\frac{\text{negative}}{\text{negative}}$) in Q1 and Q4 $\frac{\sin\theta}{\theta}$ will always be positive

in Q1 and Q4, $\cos\theta$ will always be positive (it's the x value)

$\rightarrow 1 > \frac{\sin\theta}{\theta} > \cos\theta$

finally

SQUEEZE THRM: $\lim_{\theta \to 0} 1 = 1$

$\lim_{\theta \to 0} \cos\theta = 1$ (Pamela's thrm) pm class
Tiffany's Thm in am class

because $\frac{\sin\theta}{\theta}$ is in between the two,

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ via the SQUEEZE THEOREM.

omg...