FINAL **TMath 124pm** Winter 2012

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function defined everywhere.

   T F If $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = 0$, then $\lim_{x\to\infty} [f(x) - g(x)] = \infty$.

   T F If f is continuous at x, then f is differentiable at x.

   T F $\lim_{x\to 1} \dfrac{\log_2(x)}{x-1} = \lim_{x\to 1} \dfrac{(x-1)(\log_2(x))' - (\log_2(x))(x-1)'}{(x-1)^2}$ by L'Hospital's Rule.

   T F All local extrema numbers are also critical numbers.

   T F If f has a local minimum or maximum when $x = 4$, then $f'(4) = 0$.

   T F If f is such that $f'(4)$ DNE, then there is a local minimum or maximum when $x = 4$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

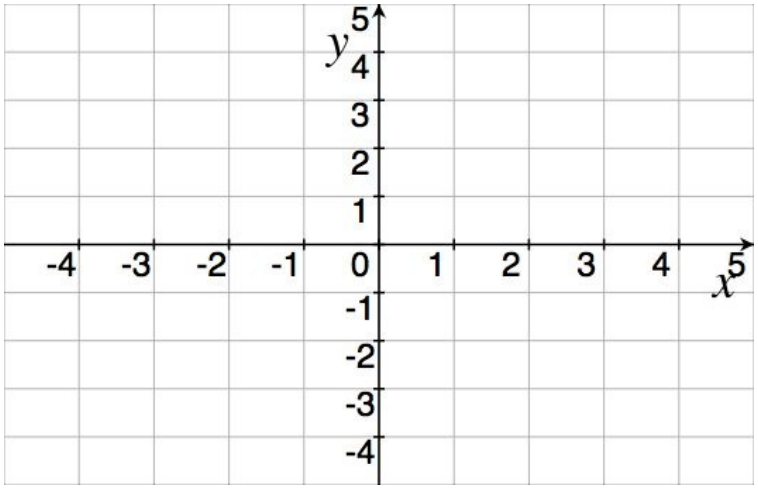
2. [7] (Exam 2 #2) Sketch a graph and then *find a formula* of an example function f that satisfies the following conditions:

   (a) f is not differentiable when $x = -4$,

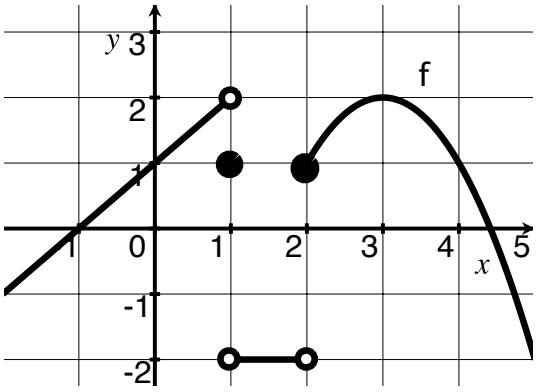
   (b) f is not continuous when $x = -4$,

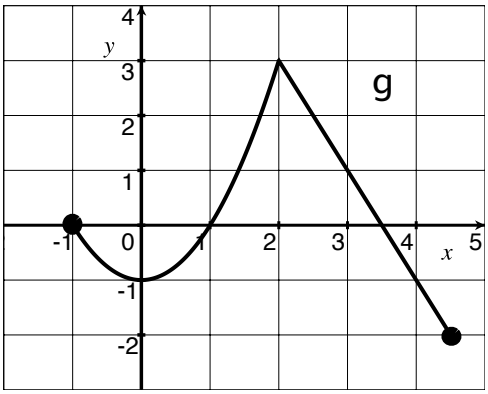
   (c) $f'(3) = -1$, and

   (d) $\lim_{x\to 1^-} f(x) = -\infty$



3. (Exam 1 #3) The graphs of f and g are shown below. Find the exact value (if possible):





$$f(x) = \begin{cases} x+1 & \text{if } x \le 1 \\ 1 & \text{if } x = 1 \\ -2 & \text{if } 1 < x < 2 \\ -(x-3)^2 + 2 & \text{if } 2 \le x \end{cases}$$

$$g(x) = \begin{cases} x^2 - 1 & \text{if } -1 \le x < 2 \\ -2x + 7 & \text{if } 2 \le x \le 4.5 \end{cases}$$

[1] (WebHW2#1)
$\lim_{x \to 1^+} f(x)$

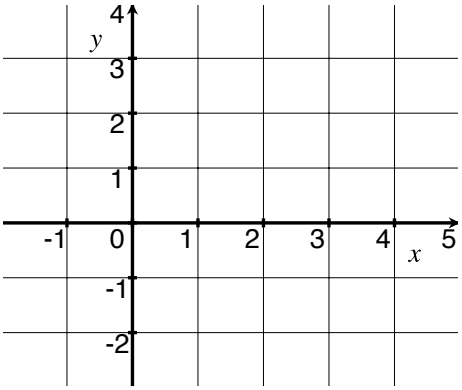
[1] (WebHW2#1)
$\lim_{x \to 2} g(x)$

[2] (§2.3 #2f)
$\lim_{x \to 3} \log_3(7 + f(x))$

[3] (Derivative Wks)
$(f \cdot g)'(4)$

[3] (§3.4 #65)
$(f \circ g)'(4)$

[3] (PracticeFinal #4) Sketch the graph of $g'(x)$ on the blank set of axes to the right.



4. Find the following *limits* if they exist. Make sure you show your work and justify your conclusions!

[3] (§2.2 Example 8)

$$\lim_{x\to 0^-} \frac{1}{x}$$

[4] Quiz 3 #1

$$\lim_{x\to 0} \frac{\sin(6x)}{\sin(2x)\cos(6x)}$$

[4] (PracticeExam1 #4)

$$\lim_{x\to -\infty} e^x \sin x$$

[3] (Limit Wks)

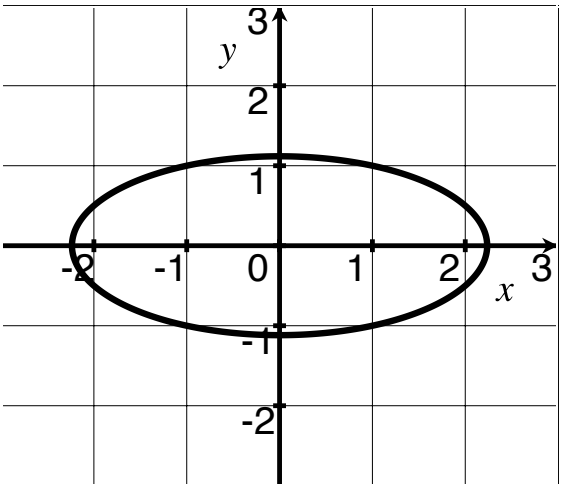
$$\lim_{x\to -2} \frac{2x^2+4x}{x+2}$$

5. Find $\frac{dy}{dx}$ for each of the following: (Do *not* simplify!)

[4] (§3.6 #47)
$y = (\sin x)^x$

[3] (PracticeExam2 #8a)
$x^2 + 4y^2 = 5$

6. The equation $x^2 + 4y^2 = 5$ defines an ellipse shown to the right.



(a) [3] (Exam 2 #7) Find the equation of the line tangent to the ellipse when $x = -1$ and $y < 0$.

(b) [4] (Derivative Wks #4) Find the points on the ellipse whose tangent lines are parallel to the line $2y + x = 4$.

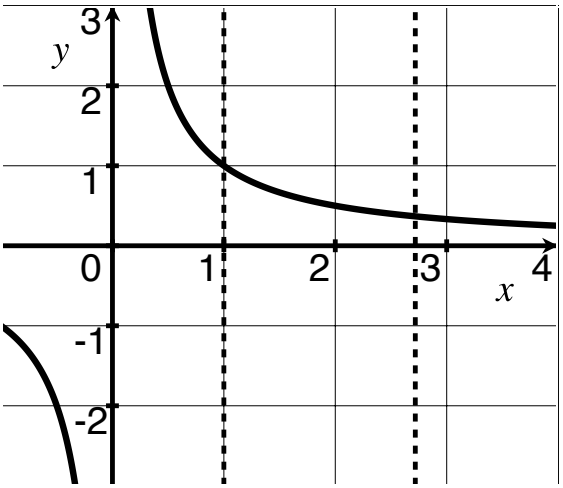
7. [3] (Quiz 5 #1) If $g(2) = 7$ and $-3 \leq g'(x) \leq 1$ for $2 \leq x \leq 5$, how small can $g(5)$ possibly be? Briefly justify your answer.

8. Find the most general antiderivative for:

   [2] (WebHW16 #1)
   $y = x - 8$

   [2] (Lecture 3/5)
   $y = 5^x \ln(5)$

9. The graph of $y = \frac{1}{x}$ is shown to the right along with the vertical lines $x = e$ and $x = 1$.



   (a) [3] (Lecture 3/5) Find $\int_1^e \frac{1}{x}\,dx$,

   (b) [1] (Lecture 3/5) Explain what you found in part (a) in terms of area.

10. [5] Choose only *ONE* of the following. Clearly identify which of the two you are answering and what work you want considered for credit.

    - (Word Wks2 #10) A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of $12\text{ft}^3/\text{min}$, how fast is the water level rising when the water is 6 inches deep?
    - (Exam2 #8) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the angle between the lader and the ground changing when the bottom of the ladder is 6ft from the wall?

11. [5] Choose only *ONE* of the following. Clearly identify which of the two you are answering and what work you want considered for credit.

    - A breeder has been selling 100 labradoodles a year at \$1500 each. A market survey indicated that for each increase in price by \$100, the number of labradoodles sold will decrease by 5 a year. Similarly for each decrease in price by \$100, the number of labradoodles sold will increase by 5 a year. Use calculus to find out what price the breeder should set so as to maximize his/her revenue?
    - (Word Wks #1) A fence 17 ft tall runs parallel to the tall building at a distance of 9 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?