Key

Exam 2 # TMath 124 Winter 2011

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function and x and y be positive numbers.

T **F** $\frac{1}{x} = \frac{1+3}{x+3}$ let $x=1$ $\frac{1}{1} \neq \frac{1+1}{1+3} = \frac{2}{4} = \frac{1}{2}$

T **F** $\lim_{x\to 0} \cos(x) = -\sin(x)$ $\frac{d}{dx}(\cos(x)) = -\sin(x)$ $\lim_{x\to 0} \cos(x) = \cos(0) = 1 \neq -\sin(x)$

T **F** $\frac{d}{dx}(e) = e$ $\frac{d}{dx}(2.7) = 0$

T F $\frac{d}{dx}(x^{-1}) = -x^{-2}$ power rule

T F $\frac{d}{dy}(y) = 1$ similarly $\frac{d}{dx}(x) = 1$

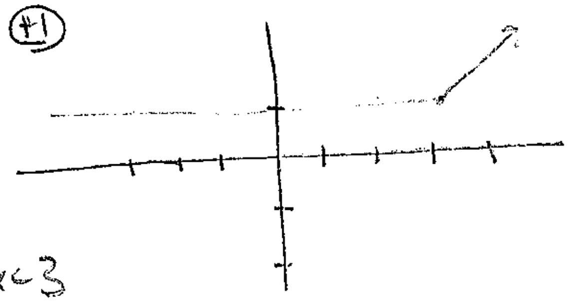
Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] Find a formula for a function f that satisfies the following criteria:

(a) f is not differentiable when $x = 3$ +1

(b) f is continuous when $x = 3$ +1

(c) $f'(-3) = 0$ +1



$$f(x) = \begin{cases} 1 & \text{if } x<3 \\ x-2 & \text{if } 3 \le x \end{cases}$$

function +.5

notation +.5

Partial: draw a graph meeting the conditions +.5 each + .5 function notation (+.5)

Sew $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

(+.5)

alg (+.5)

3. [3] (Quiz 3 #1) Find the *limit*: $\lim_{x\to 0} \frac{\sin(6x)}{\cos(6x)\sin(2x)}$

nota (+.5) nota (+.5)

$$\lim_{x\to0} \frac{\sin 6x}{1}\left(\frac{6x}{6x}\right)\cdot\frac{1}{\cos 6x}\cdot\frac{1}{\sin(2x)}\left(\frac{2x}{2x}\right)$$

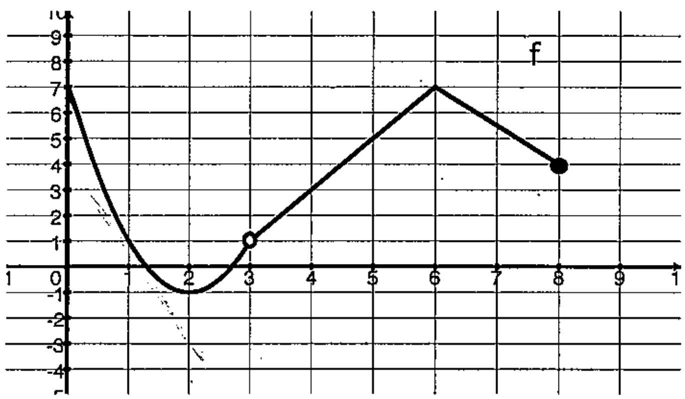
$$= \lim_{x\to0} \frac{\sin 6x}{6x}\cdot 6x\,\frac{1}{\cos 6x}\cdot\frac{2x}{\sin 2x}\cdot\frac{1}{2x}$$

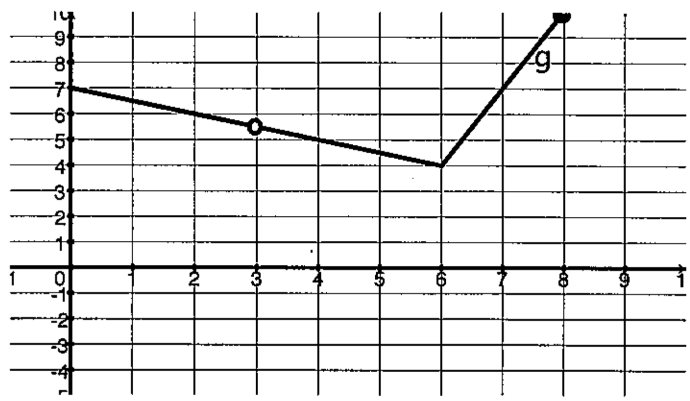
$$\lim_{x\to0}\frac{6x}{2x\cos 6x}$$

$$= \lim_{x\to0}\frac{3}{\cos 6x} = \frac{3}{\cos(0)}$$

$= 3$ limit prop (.5)

4. [8] Let the graph of f and g be those shown below.





Estimate the following (if they exist):

[2] (§3.2 #44)
$(5f - 4g)'(1)$

$5f'(1) - 4g'(1)$ (+.5)

(+.5) (+.5)

notation/alg (+.5)

$5(-4) - 4(-\tfrac{1}{2})$

$-20 + 2 = -18$

[2] (DerivativeWks)
$(f \cdot g)'(4)$

product rule (+.5)

$f(4)g'(4) + g(4)f'(4)$

(+.5) (+.5) (+.5)

$3\cdot(-\tfrac{1}{2}) + 5\cdot 2$

$-\tfrac{3}{2} + 10 = 8.5$

[3] (§3.4 #65)

$(f \circ g)'(4)$ Chain rule (+1)

$f'(g(4))\,g'(4) = f'(5)\cdot(-\tfrac{1}{2})$

(+.5) (+.5) (+.5)

$= 2\cdot -\tfrac{1}{2}$

$= -1$

notation (.5)

[3] (WebHW8 #10)

$\left(\frac{f}{g}\right)'(3)$ quotient (+1)

(+1) notation/alg

$$\frac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2}$$

$g'(3)$ & $f'(3)$ do not exist (+1)

$\Rightarrow (f/g)'(3)$

does not exist.

5. [14] Find the derivatives of the following and do *not* simplify.

[3] (WebHW8 #1)

$y = \underbrace{5e^x}_{f}\underbrace{\sqrt{x}}_{g}$ product +1

$f(x)g'(x) + g(x)f'(x)$

$[5e^x]\frac{d}{dx}(x^{1/2}) + x^{1/2}\frac{d}{dx}[5e^x]$

$5e^x \underbrace{\frac{1}{2}\cdot x^{-1/2}}_{+.5} + x^{1/2}\underbrace{5}_{+.5}\frac{d}{dx}[e^x]$

$\frac{5}{2}e^x x^{-1/2} + 5x^{1/2}\underbrace{e^x}_{+.5}$

notation +.5

[3] (PracticeExam #6)

$y = \dfrac{\sin(x) + x^2\cos(x)}{\cos(x)}$ notation +.5   derivative +.5

$y = \dfrac{\sin(x)}{\cos(x)} + \dfrac{x^2\cos(x)}{\cos(x)} = \tan(x) + x^2$

alg +1

$dy/dx = \underbrace{\sec^2(x)}_{+.5} + \underbrace{2x}_{+.5}$

or

$y = \dfrac{\cos(x)[\sin(x) + x^2\cos(x)]' - [\sin(x) + x^2\cos x](\cos(x))'}{[\cos x]^2}$ +1

$= \dfrac{\cos(x)\overbrace{[\cos x + [x^2(-\sin x) + 2x\cos x]]}^{+1} + [\sin(x) + x^2\cos x]\sin}{\cos^2 x}$

notation +.5 $d/dx(\cos x) = -\sin x$ +.5

[4] (DiffPractice Wks)

$y = x^2y^3 + x^3y^2$

$\frac{d}{dx}(y) = \frac{d}{dx}[x^2y^3 + x^3y^2]$

$dy/dx = \frac{d}{dx}[x^2y^3] + \frac{d}{dx}[x^3y^2]$

product +1

$dy/dx = x^2\frac{d}{dx}[y^3] + y^3\frac{d}{dx}[x^2] + x^3\frac{d}{dx}[y^2] + y^2\frac{d}{dx}[x^3]$

$dy/dx = x^2\,3y^2\,dy/dx + y^3\,2x + x^3\,2y\,dy/dx + y^2\,3x^2$

+1   +.5

$dy/dx - x^2 3y^2\,dy/dx - 2x^3y\,dy/dx = 2xy^3 + 3y^2x^2$ alg +1

tried to solve for dy/dx +.5

$dy/dx\,[1 - 3x^2y^2 - 2x^3y] = 2xy^3 + 3y^2x^2$

$dy/dx = \dfrac{2xy^3 + 3y^2x^2}{1 - 3x^2y^2 - 2x^3y}$

[4] (§3.4 #24)

$y = 10^{1-x^2}$ Chain +1

$g(x) = 1 - x^2$ $g'(x) = -2x$

$\underbrace{f(u) = 10^u}_{+.5}$ $\underbrace{f'(u) = 10^u\ln 10}_{+1}$

$f'(g(x))g'(x)$

$= f'(1-x^2)\cdot(-2x)$

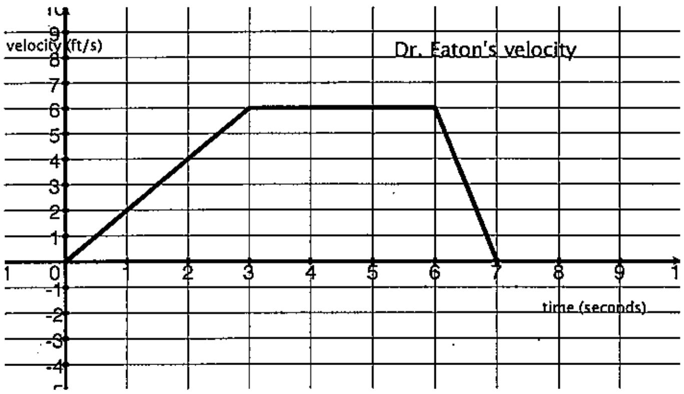
$= 10^{1-x^2}(\ln 10)(-2x)$

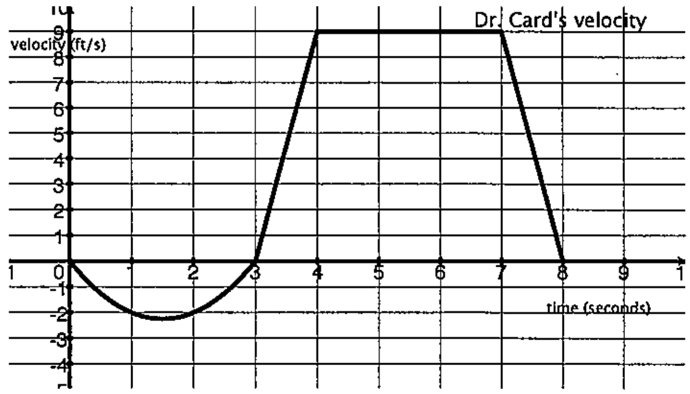
$= -2(\ln 10)\times 10^{1-x^2}$

plug in +1

notation +.5

6. (Word Problem Wks #4) Dr. Card and Dr. Eaton decide to have a short race. The following is a graph of their respective *velocities* at time t measured in seconds.





(a) [2] Who can run faster? What is his/her top speed?

Dr Card, (+1) top speed of 9 (+1)

(b) [2] Both are stationary at the start of the race. What other times are Dr. Card & Dr. Eaton at rest?

Dr Eaton is at rest after 7 sec (+.5)

Dr Card is at rest after 3 sec and 8 sec. (+.5) (-1)

(c) [1] When is Dr. Card running the wrong way?

(0,3)

(d) [1] When is Dr. Eaton decelerating?

(6,7)

7. [3] (Exp Wks #2) Consider $f(x) = e^x + \frac{1}{x}$. Find the equation of the line tangent to the graph of f when $x = 1$.

Looking for $y = mx + b$ (+.5)

(+.5) m = slope of line tangent to f when $x=1$ $= f'(1)$

$f'(x) = [e^x + 1/x]'$

$= [e^x]' + [x^{-1}]'$

$= e^x - x^{-2}$ (+.5)

$\Rightarrow f'(1) = e^1 - 1^{-2} = e - 1$ (+.5)

Notice the line tangent to f shares the point $(1, f(1))$ with the graph of f. $(1, f(1)) = (1, e^1 + \frac{1}{1})$

$= (1, e^1 + 1)$ (+.5)

So

$y - (e+1) = (e-1)[x-1]$ plug in (+.5)

$y = ex - x - \cancel{e} + 1 + \cancel{e} + 1$

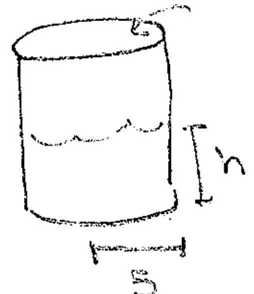
$y = (e-1)x + 2$

8. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (Story Problem Worksheet #3) A cylindrical tank with radius 5m is being filled with water. The height of the water seems to be increasing at a rate of 5cm/min. Find the rate that water is being added to the tank.

(b) (§3.9 Example 2) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the angle between the lader and the ground changing when the bottom of the ladder is 6ft from the wall?

a)



$Vol = \pi r^2 h = \pi 5^2 h = \underbrace{25\pi h}_{(+.5)}$

$\Rightarrow \frac{dVol}{dt} = \frac{d}{dt}[25\pi h]$

$\Rightarrow \underbrace{\frac{dVol}{dt} = 25\pi \frac{dh}{dt}}_{(+1)}$

started (+.5)
notation (+.5)
picture (+.5)
alg (+.5)

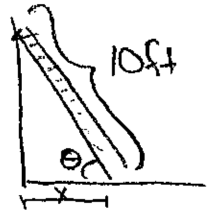
notice we were given

$\underset{(+.5)}{dh/dt = 5\ cm/min} \cdot \frac{1m}{100cm} = .05\ m/min$ units (+.5)

$\Rightarrow \frac{dVol}{dt} = 25\pi \cdot .05 = 3.925\ m^2/min$ plug in given (+.5)

$m^2 \quad m/min$

b)



$dx/dt = 1\ ft/s$

Sohcahtoa

$\cos\theta = \frac{x}{10}$

$\underbrace{10\cos\theta = x}_{(.5)}$

notice we are given

$dx/dt = 1\ ft/s$ (+.5)

$\Rightarrow \frac{d}{dt}(10\cos\theta) = \frac{d}{dt}(x)$

$10\frac{d}{dt}(\cos\theta) = \frac{dx}{dt}$

$10(-\sin\theta)\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$

$\underbrace{-10\sin\theta\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}}_{(+1)}$

started (+.5)
notation (+.5)
picture (+.5)
alg (+.5)

Want $\left.\frac{d\theta}{dt}\right|_{x=6ft}$

(+.5) when $x=6$ opp, hyp 10, adj 6, $\sin\theta = \frac{opp}{10}$

opp $= \sqrt{10^2-6^2} = \sqrt{100-36} = \sqrt{64} = 8$

$\sin\theta = 8/10 = 4/5$

plug in given (+.5)

$\Rightarrow -10\left(\frac{4}{5}\right)\left.\frac{d\theta}{dt}\right| = 1 \Rightarrow -8\frac{d\theta}{dt} = 1 \Rightarrow \left.\frac{d\theta}{dt}\right|_{x=6} = -\frac{1}{8}$