

TMATH 124am: Quiz 3

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. There are two sides of this quiz.

1. [2] (Trig Wks #2b) Find the limit: $\lim_{x \rightarrow 0} \frac{\sin(3x) \sin(5x)}{x^2}$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{\sin(3x) \sin(5x)}{x^2} \quad \left(\frac{3.5}{3.5} \right) \quad \text{notation } \textcircled{+5} \\
 & = \lim_{x \rightarrow 0} 15 \frac{\sin 3x}{3x} \cdot \frac{\sin 5x}{5x} \quad \text{alg } \textcircled{+5} \\
 & = \lim_{x \rightarrow 0} 15 \cdot \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \quad \text{see } \textcircled{+5} \\
 & = 15 \cdot 1 \cdot 1 = \textcircled{15} \quad \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1
 \end{aligned}$$

note: incorrect notation for derivatives will be identified in the marking but not penalized until the exam

2. (§3.1 #35) Let $f(x) = x^4 + 2e^x$.

(a) [1] Find $f'(x)$.

$$f'(x) = [x^4 + 2e^x]' = [x^4]' + [2e^x]' = [x^4]' + 2[e^x]'$$

$$= \underbrace{4x^3}_{+1.5} + \underbrace{2e^x}_{+1.5}$$

(b) [3] Find the equation of the line that is tangent to f at the point $(0, 2)$.

Looking for $y = mx + b$ (+.5)

$m =$ slope of line tang. $= f'(0)$ (+1)
to f when $x=0$

$$= 4(0)^3 + 2 \cdot e^0 = 2 \quad (+.5)$$

(+.5) { the line is tang. to f at $(0, 2)$ so it passes through the point $(0, 2)$
 $\Rightarrow d = 2(0) + b$
 $\Rightarrow b = 2$

Thus $y = 2x + 2$

3. (WebHW7 #9) Let $g(x) = x + \cos(x)$.

(a) [1] Find $\frac{d}{dx}g(x)$.

$$\frac{d}{dx}g(x) = \frac{d}{dx}[x + \cos(x)] = \frac{d}{dx}(x) + \frac{d}{dx}(\cos(x))$$

$$= 1 - \sin(x)$$

(b) [3] Find all x values where the graph of g has a horizontal tangent line.

We want to find the x values so the slope of the line tang. to g at x equals 0

$$g'(x) = 0$$

alg (+.5)

Find x when

$$0 = 1 - \sin x$$

$$\Rightarrow -1 = -\sin x$$

$$\Rightarrow 1 = \sin x$$

happens at angles coterminal with $\frac{\pi}{2}$ (+.5)

so $\frac{\pi}{2}, \frac{\pi}{2} + 2\pi, \frac{\pi}{2} + 4\pi, \dots$

or $\frac{\pi}{2} + n2\pi$ where n is an integer (+.5)