TMath 124

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions and x and y be positive numbers.

   T   F $\dfrac{2x}{6x+y} = \dfrac{x}{3x+y}$

   T   F $\dfrac{x^2+x-6}{x-2} = x+3$

   T   F $\lim\limits_{x\to a}(f(x)+g(x)) = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$ where a is in the domain of f and g.

   T   F   If $f'(a)$ exists, then $\lim\limits_{x\to a} f(x) = f(a)$.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [6] (§2.2 #16, §2.5 #8, & §2.7 #21) Sketch the graph of an example function f that satisfies the following conditions:

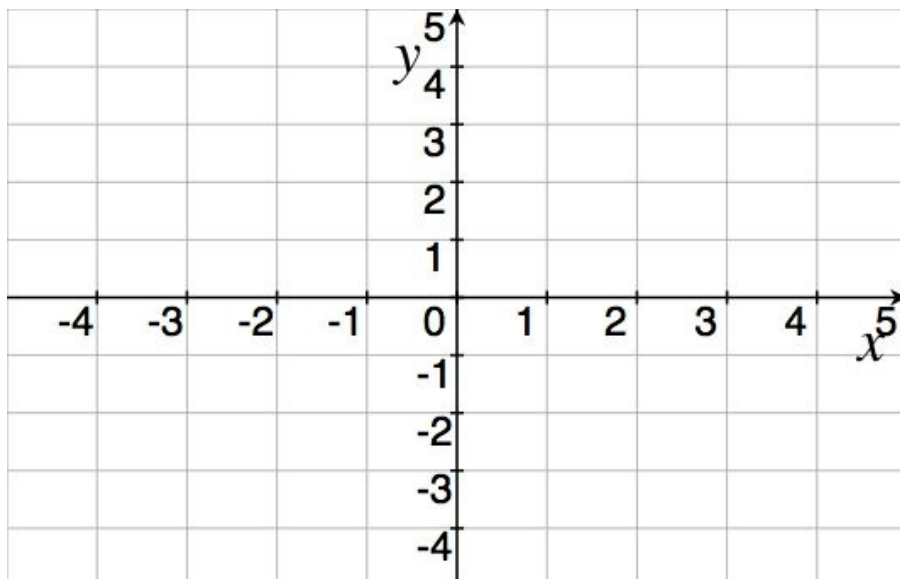   (a) f is continuous everywhere but when $x=2$ and $x=4$.

   (b) $f(2) = 4$
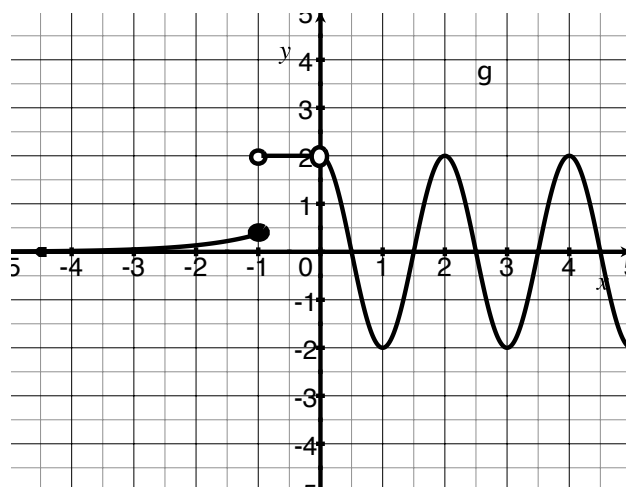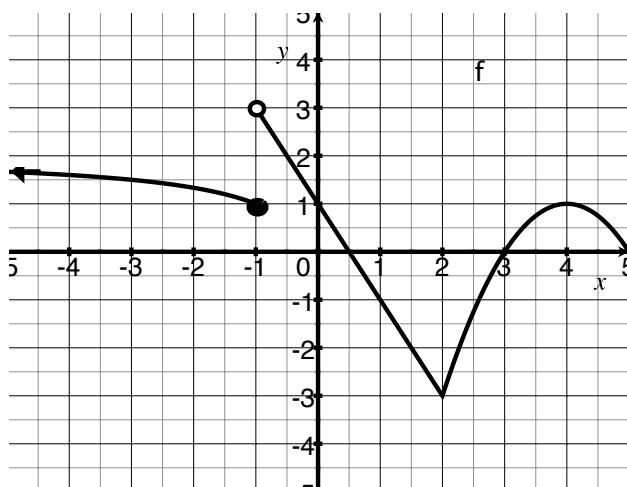
   (c) $\lim\limits_{x\to 2} f(x) = -3$

   (d) $\lim\limits_{x\to 4^+} f(x) = -\infty$

   (e) $f(-1) = 2$

   (f) $f'(-1) = 0$

3. [14] The graphs of f and g and their rules are provided below:

$$f(x) = \begin{cases} \frac{2x}{x-1} & \text{if } x \le 0 \\ -2x + 1 & \text{if } -1 < x \le 2 \\ -(x-4)^2 + 1 & \text{if } 2 < x \end{cases} \qquad g(x) = \begin{cases} e^x & \text{if } x \le -1 \\ 2 & \text{if } -1 < x < 0 \\ 2\cos(\pi x) & \text{if } 0 < x \end{cases}$$
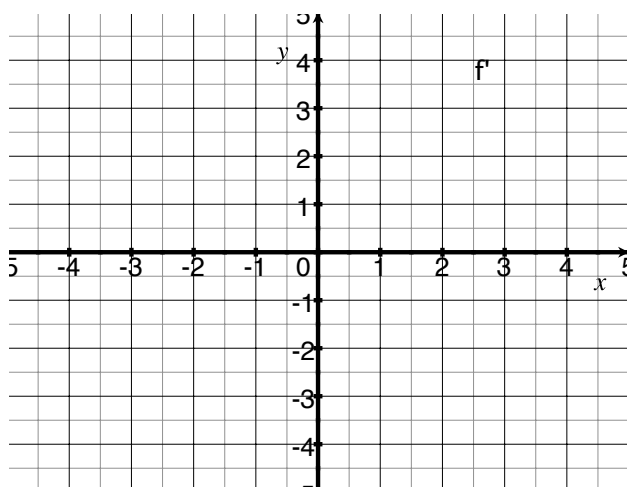
Use the graphs above to find the following (if they exist!):

[2] (WebHW2 #3) $\displaystyle\lim_{x \to -1^-} g(x)$

[2] (WebHW5 #1) $\displaystyle\lim_{x \to -\infty} f(x)$

[3] (§2.3 #2) $\displaystyle\lim_{x \to 0}[6g(x) - f(x)]$

[3] (WebHW5 #9) $f'(1)$

[4] (§2.8 #9) Make a rough
sketch of the graph of $f'(x)$:

4. [12] Find the limit or explain why it does not exist.

(WebHW4 #4) $\lim\limits_{x \to 9} \dfrac{40 + \sqrt{x}}{\sqrt{40 + x}}$

(WebHW3 #7) $\lim\limits_{x \to -5} \dfrac{\frac{1}{5} + \frac{1}{x}}{5 + x}$

(InfLimits Wks) $\lim\limits_{x \to -\infty} \dfrac{x^7 + x}{x^5 - 4}$

(Practice #4) $\lim\limits_{x \to \infty} e^{-2x} \sin(x)$

5. [3] (CalcWebHW5 #10) Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 6$ if $g(6) = -2$ and $g'(6) = 3$.

6. Let $f(x) = x^2 - x$.

   (a) [2] (§3.1 #17) Find $f'(x)$

   (b) [4] (§2.8 #29) Find the derivative of f using the definition of derivative. That is, use $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ and verify your answer to part (a).

7. [5] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

(a) A tank contains 5000L of pure water. Brine that contains 30 g of salt per liter of water is pumped into the tank at a rate of 25L/min. Find a function that records the concentration of salt after t minutes (in grams per liter) and then find out what happens to the concentration as $t \to \infty$.

(b) Explain how scientists know there are at least two points directly opposite each other on the surface of the earth that are the same temperature.