TMATH 124: Quiz 5 Key

You may use any work of yours that you made from last week. This includes, practice book problems and worked out WebAssign problems. This *does not* include photocopies of notes from the book or tutorials shown on WebAssign. Graphing calculators are also not allowed. In short, you are only allowed to use *work* that you created.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [2 each] Differentiate each of the following. Please do *not* simplify for this problem.

$y = \log_2(1 - 3x)$

$f(x) = \log_2 x$ $\quad f'(x) = \frac{1}{x \ln 2}$ (+.5)

$g(x) = 1 - 3x$ $\quad g'(x) = -3$ (+.5)

chain (+.5)

$y' = f'(g(x))\, g'(x)$

$= \frac{1}{(1-3x)\ln 2} \cdot -3$ } got it (+.5)

$y = x^x$

$\ln y = \ln x^x$

$\Rightarrow \ln y = x \ln x$

introduce ln (+.5)

pop ln (+.5)

product (+.5)

$\frac{d}{dx}$ (

$\frac{1}{y} \cdot y' = x(\ln x)' + (x)' \ln x$

$\frac{1}{y} \cdot y' = x \cdot \frac{1}{x} + 1 \cdot \ln x$

$y' = y(1 + \ln x)$ or $x^x(1 + \ln x)$

get it (+.5)

2. [3] Find an equation of the tangent line to the curve $y = x^2e^{-x}$ when $x = 1$.

2 ways: original

$y' = x^2(e^{-x})' + (x^2)'e^{-x}$ product +1.5

$y' = -x^2e^{-x} + 2xe^{-x}$ got it +1.5

So $m = y'(1) = -1^2e^{-1} + 2(1)e^{-1} = \frac{-1}{e} + \frac{2}{e}$ +1.5

$= \frac{1}{e}$

logarithmic

$\ln y = 2\ln x + \ln e^{-x}$ logs

$\ln y = 2\ln x - x$ ln prop +1.5

$\frac{d}{dx}$ ( ) der +1.5

$\frac{1}{y} \cdot y' = \frac{2}{x} - 1 \Rightarrow y' = y\left(\frac{2}{x} - 1\right)$ +1.5

So $m = y'(1) = y\left(\frac{2}{1} - 1\right) = (1)^2e^{-1}(1) = \frac{1}{e}$

Looking for $y = mx + b$ from the above work we have $m = 1/e$ +1.5

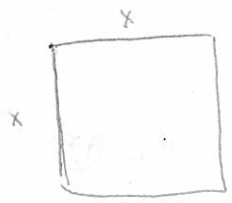
The line passes through $(1, (1)^2e^{-1}) = (1, 1/e)$ +1.5

$\frac{1}{e} = \frac{1}{e} \cdot (1) + b$

$\Rightarrow b = 0$ +1.5

So $y = \frac{1}{e} \cdot x$

3. [3] Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?



$\frac{dx}{dt} = 6$ cm/s

variables +1.5

$A = x^2$

connect x to A +1.5

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

took der +1.5

found what x was when A was 16 +1.5

$\left.\frac{dA}{dt}\right|_{\text{area}=16} = 2 \cdot 4 \cdot 6 = 48$ got it +1.5