# Math 124

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let $f$ and $g$ be differentiable functions and $h$ be a constant.

   T   F   $\frac{x+h}{2x} = \frac{1+h}{x}$

   T   F   $\sqrt{x^2 + h^2} = x + h$

   T   F   $\lim_{x \to r} f(x) = f(r)$ for all $r$ in the domain of $f$.

   T   F   If $\lim_{x \to r} g(x) = 0$, then $\lim_{x \to r} \frac{f(x)}{g(x)}$ does not exist.

   T   F   $\frac{d}{dx}\left(\frac{1}{x}\right) = -1$

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).
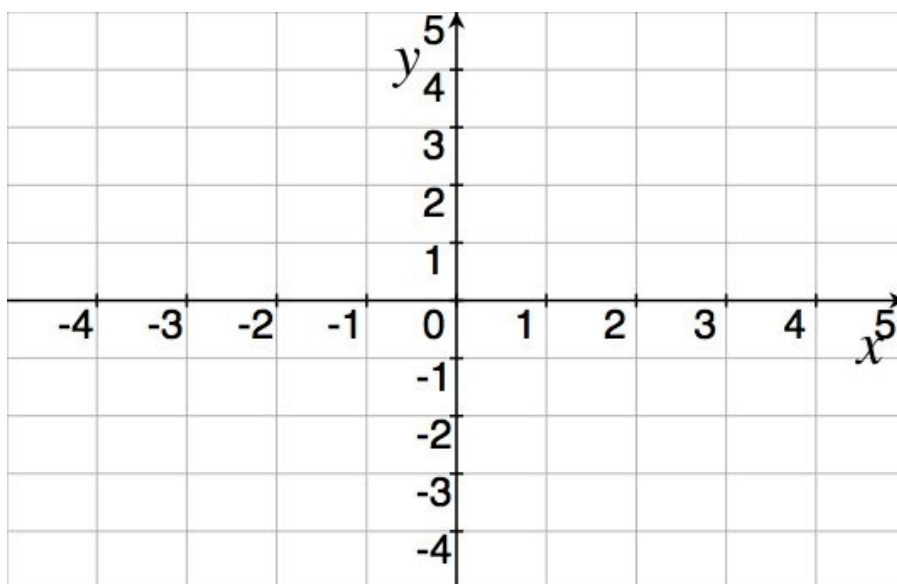
2. [5] Sketch the graph of an example function $f$ that satisfies the following conditions:

   (a) $f$ is not differentiable when $x = -3$

   (b) $f$ is continuous when $x = -3$

   (c) $\lim_{x \to 1^+} f(x) = \infty$

   (d) $f'(3) = -2$

3. Compute the following limits:

(a) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{2x^2 - 8x + 6}$

(b) $\lim\limits_{x \to \infty} \dfrac{x^2 + x - 2}{2x^2 - 8x + 6}$

(c) $\lim\limits_{\theta \to 0^+} \dfrac{\theta + \theta^2}{1 - \cos \theta}$

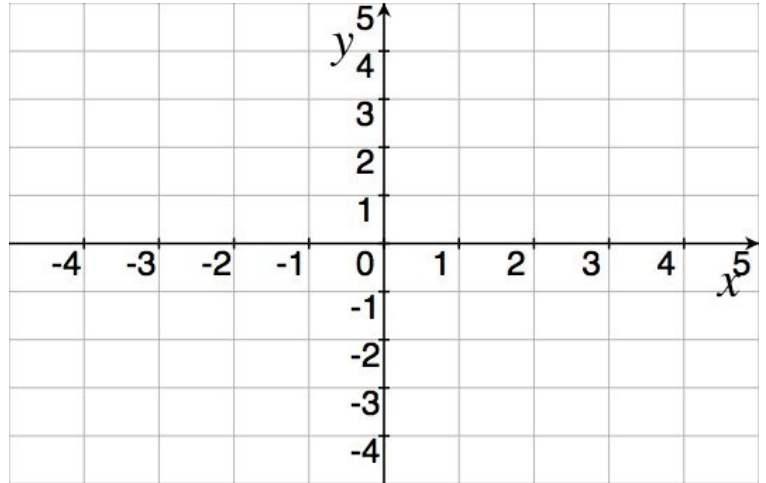(d) $\lim\limits_{x \to \infty} x \sin\left(\dfrac{5\pi}{x}\right)$

(e) $\lim\limits_{x \to 0} x^4 \sin\left(\dfrac{1}{x}\right)$

(f) $\lim\limits_{x \to 1} \dfrac{1}{x - 1}$

4. Let $f(x) = \begin{cases} \sqrt{1-(x+3)^2} & \text{if } -4 \le x \le -2 \\ 1 & \text{if } -2 < x < 2 \\ -(x-3)^2 + 2 & \text{if } 2 < x < 4 \\ -3 & \text{if } 4 < x \end{cases}$

Graph $f(x)$ and then graph $f'(x)$ below on its own set of axes. Afterwards, answer the following questions.
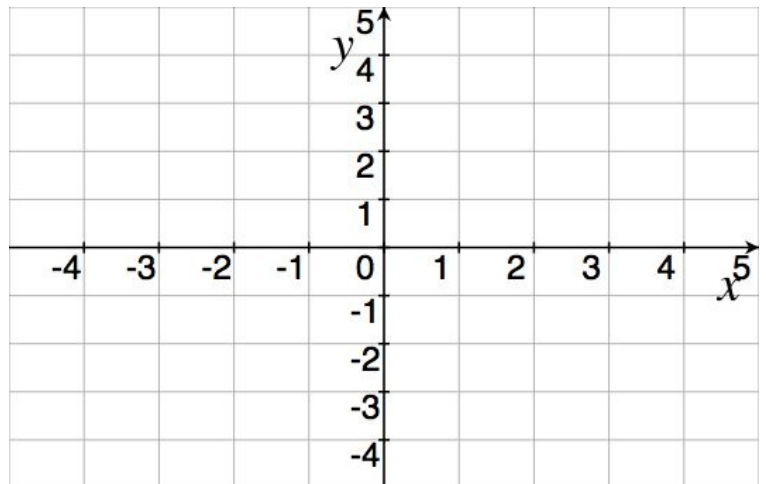
(a) $\lim\limits_{x \to 2} f(x)$

(b) $\lim\limits_{x \to -2^+} f(x)$

(c) $\lim\limits_{x \to 3} f'(x)$

(d) $\lim\limits_{x \to \infty} f(x)$

5. Compute the derivatives of the following functions. You do *not* need to simplify.

(a) $f(x) = x^3 + 3^x + \pi^\pi$

(b) $g(t) = \ln(t) \left( \dfrac{2 + t^2}{3t - 1} \right)$

(c) $h(\theta) = 7 \sec(\sqrt{\theta})$

(d) $y = \sqrt{x} e^{x^7} (x^6 + 3)^{10}$

(c) $y = (\cos(x))^x$

(d) $x^2 y^2 = 4 - y \arctan(5x)$

6. Find the equation of the line tangent to the graph of $f$ when $x = 2$ if $f(x) = m(n(x))$, $n(2) = -1$, $m(-1) = 6$, $n'(2) = 3$, and $m'(-1) = 5$.

7. Find the antiderivative for each of the following functions:

(a) $2x - x^3 + 7\sin(x)$

(b) $\dfrac{5 - 4x^3 + 2x^6}{x^6}$

8. Consider the function $f(x) = \sqrt[3]{x}$

(a) Evaluate the integral $\displaystyle\int_1^8 \sqrt[3]{x}\, dx$

(b) Draw a picture that corresponds to the area you computed in (a).

9. A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2\text{m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3m deep.

10. A street light is mounted at the top of a 13 ft pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 30 ft away from the pole?

11. Find the dimensions of the rectangle of largest area that has its base on the $x$-axis and its other two vertices above the $x$-axis and lying on the parabola $y = 7 - x^2$

12. A truck has a minimum speed of 9 mph in high gear. When traveling x mph, the truck burns diesel fuel at the rate of

$$0.003935 \left( \frac{675}{x} + x \right) \frac{\text{gal}}{\text{mile}}$$

Assume that the truck can not be driven over 63 mph, that diesel fuel costs $2.84 a gallon, and that the driver is paid $12 an hour. Find the speed that will minimize the cost of a 500 mile trip.