Optimization

part two

1. An industrial production process costs $C(q)$ million dollars to produce q million units; these units then sell for $R(q)$ million dollars. If $C(2.1) = 5.1$, $R(2.1) = 6.9$, $MC(2.1) = 0.6$, and $MR(2.1) = 0.7$, find the following:

   (a) The profit earned by producing 2.1 million units.

   \$ in − \$ out $= 6.9 - 5.1 = 1.8$ million dollars

   (b) Should the company increase or decrease production to maximize profit?

   The company makes more \$ (70 cents) than it costs (60 cents) so that last item made is increasing our overall profit.

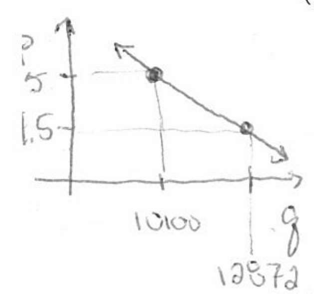
2. Production of an item has fixed costs of \$10,000 and each item costs \$2 to produce. Assume the relationship between price (p) and quantity demanded (q) is *linear*. Market research shows that 10,100 items are sold when the price is \$5 and 12,872 items are sold when the price is \$4.50.

   (a) Express the cost, C, of producing q items.

   fixed costs + costs of producing q million units

   $10{,}000 + 2 \cdot q$

   (b) Recall that the demand curve is *linear*. Express p, as a function of q.



   Looking for $y = mx + b$ or rather $p = mq + b$

   $m = \frac{\text{rise}}{\text{run}} = \frac{5 - 4.50}{10100 - 12872}$

   $= \frac{-.5}{2772} = \frac{-1}{5544}$

   line passes thru $(10{,}100, 5)$ so

   $5 = \left(\frac{-1}{5544}\right)(10100) + b$

   $\rightsquigarrow b = 6.82$

   So $p = \frac{-1}{5544} q + 6.82$

   (c) Recall if you sell q items for \$$p$, then you will have $p \cdot q$ dollars of revenue. Use the work from (b) to express the revenue, R, from selling q items as *only* a function of q.

   Revenue $= (p)\, q$

   $= \left(\frac{-1}{5544} q + 6.82\right) q$

   (d) Express the profit earned as a function of q. Use this to find how many items the company should produce to maximize profit.

Profit = (Revenue) − (Costs)

$= \left(\left[\frac{-1}{5544} q + 6.82\right] q\right) - (10{,}000 + 2q)$

$= \frac{-1}{5544} q^2 + 4.82q - 10{,}000$

note: the profit function is a parabola opening down thus the extrema will be a maximum.

$P'(q) = \frac{-2}{5544} q + 4.82$

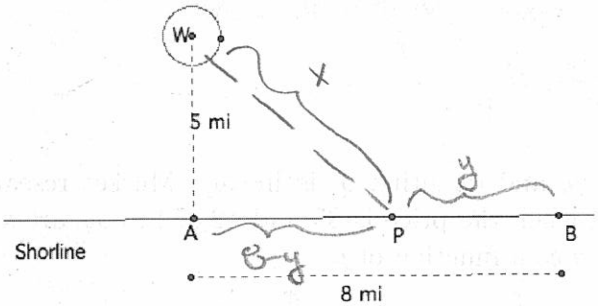
Finding the critical points

$0 = \frac{-2}{5544} q + 4.82$

$q = -4.82\left(-\frac{5544}{2}\right)$

$\approx 13{,}389$ units produced will maximize profit

2. An offshore oil well is located in the ocean at a point W, which is 5 miles from the closest shorepoint A on a straight shoreline. The oil is to be piped to a shorepoint B that is 8 miles from A by piping it on a straight line under water from W to some shorepoint P between A and B ~~ad~~ and then on to B via a pipe along the shoreline. If the cost of laying pipe is \$100,000 per mile under water and \$75,000 per mile over land, where should the point P be located to minimize the cost of laying the pipe?



Cost of pipe = Cost under water + cost on land

= 100,000x + 75000y

need to get this in terms of one variable.

by (*)

= 100,000($\sqrt{25+(8-y)^2}$) + 75000y

We'll need to find CP & then find min

$5^2 + (8-y)^2 = x^2$

$\Rightarrow x = \sqrt{25+(8-y)^2}$ (*)

$\text{Cost}'(x) = 100{,}000\ \cancel{\tfrac{1}{2}}(25+(8-y)^2)^{-1/2}(\cancel{2}(8-y)(-1)) + 75000$

$= -100{,}000(8-y)(25+(8-y)^2)^{-1/2} + 75000$

CP Cost'(y) = 0 or Cost'(y) is undef $\leadsto$ y = 2.33 so place P 2.33 mi to the left of B.

− + 2.33

3. You run a small furniture business. You sign a deal with a customer to deliver up to 400 chairs, the exact number to be determined by the customer later. The price will be \$90 per chair up to 300 chairs, and above 300, the price will be reduced by \$0.25 per chair (on the whole order) for every additional chair over 300 ordered.

(a) Write down the revenue as a function of number of chairs sold q. Note, you might want a piece-wise defined function here.

$$\text{Revenue} = \begin{cases} 90x & \text{if } x \le 300 \\ [90 - .25(x-300)]\,x & \text{if } x > 300 \end{cases} = \begin{cases} 90x & ; x \le 300 \\ -.25x^2 + 165x & ; x > 300 \end{cases}$$

(b) What is the largest and smallest revenues your company can make under this deal?

First function is linear $\Rightarrow$ no critical points but at endpoints (ie when x = 0 or 300)

Second function has a critical point when $-.5x + 165 = 0 \leadsto x = 330$

+ − 330 local max

if x = 0 Rev is 0 smallest

if x = 300 Rev is 27000

if x = 330 Rev is 27,225 ← largest