

(needs to be solved)

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Implicit Differentiation

Key

1. Assume that y is a function of x . Find $\frac{dy}{dx}$ in the following:

(a) $x^3 + y^3 = 8$

$$\begin{aligned}\frac{\partial}{\partial x}(x^3 + y^3) &= \frac{\partial}{\partial x}(8) \\ \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial x}(y^3) &= 0 \\ 3x^2 + \frac{\partial}{\partial x}(y^3) &= 0\end{aligned}$$

$$\left. \begin{aligned} \frac{\partial}{\partial x}(y^3) &= f'(y) \\ f(x) &= x \\ f'(x) &= 3x^2 \\ g(x) &= y \\ g'(x) &= \frac{dy}{dx} \\ \frac{\partial}{\partial x}(y^3) &= f'(y) \cdot \frac{dy}{dx} = 3y^2 \frac{dy}{dx} \end{aligned} \right\}$$

$$\begin{aligned} 3x^2 + 3y^2 \frac{dy}{dx} &= 0 \\ 3y^2 \frac{dy}{dx} &= -3x^2 \\ \frac{dy}{dx} &= \frac{-3x^2}{3y^2} = \frac{x}{y} \end{aligned}$$

(b) $y = x^2y^3 + x^3y^2$

$$\begin{aligned}\frac{\partial}{\partial x}(y) &= \left[\frac{\partial}{\partial x}(x^2y^3) + \frac{\partial}{\partial x}(x^3y^2) \right] \\ \frac{dy}{dx} &= [x^2 \frac{\partial}{\partial x}(y^3) + \frac{\partial}{\partial x}(x^3)y^3] + [x^3 \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial x}(x^3)y^2] \\ \frac{dy}{dx} &= x^2 3y^2 \frac{dy}{dx} + 2xy^3 + x^3 2y \frac{dy}{dx} + 3x^2y^2 \\ \frac{dy}{dx} - x^2 3y^2 \frac{dy}{dx} - x^3 2y \frac{dy}{dx} &= 2xy^3 + 3x^2y^2\end{aligned}$$

(c) $y = \sin(2x + 5y)$

$$\begin{aligned}\frac{\partial}{\partial x}(y) &= \frac{\partial}{\partial x}(\sin(2x + 5y)) \\ \frac{dy}{dx} &= \cos(2x + 5y)[2 + 5 \frac{dy}{dx}]\end{aligned}$$

$$\frac{dy}{dx} = 2\cos(2x + 5y) + 5 \frac{dy}{dx} \cos(2x + 5y)$$

$$\frac{dy}{dx} - 5 \frac{dy}{dx} \cos(2x + 5y) = 2\cos(2x + 5y)$$

(d) $e^{xy} = e^{3x} - e^{4y}$

$$\Rightarrow \frac{dy}{dx} = \frac{2\cos(2x + 5y)}{1 - 3x^2y - 2x^3y}$$

$$\frac{\partial}{\partial x}(e^{xy}) = \frac{\partial}{\partial x}(e^{3x}) - \frac{\partial}{\partial x}(e^{4y}) \Rightarrow \frac{dy}{dx} = \frac{2\cos(2x + 5y)}{1 - 5\cos(2x + 5y)}$$

$$\begin{aligned}f(x) &= e^x & g(x) &= xy & f(x) &= e^x & g(x) &= 3x & F(x) &= e^x & G(x) &= 4y \\ f'(x) &= e^x & g'(x) &= x \frac{dy}{dx} + y & f'(x) &= e^x & g'(x) &= 3 & F'(x) &= e^x & G'(x) &= 4 \frac{dy}{dx}\end{aligned}$$

$$e^{xy} \left(x \frac{dy}{dx} + y \right) = e^{3x} \cdot 3 - e^{4y} \cdot 4 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} x e^{xy} + 4 e^{4y} \frac{dy}{dx} = 3 e^{3x} - 4 e^{4y} \frac{dy}{dx}$$

$$\frac{dy}{dx} x e^{xy} + y e^{xy} = 3 e^{3x} - 4 e^{4y} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{3 e^{3x} - 4 e^{4y} y e^{xy}}{x e^{xy} + 4 e^{4y}}$$

2. Let C be the graph of a circle centered at $(1, 0)$ with radius 3

- (a) Write down the equation of the circle C that you are working with.

$$(x-1)^2 + (y-0)^2 = 3^2 \quad \text{so} \quad (x-1)^2 + y^2 = 9$$

- (b) Find the equation of line tangent to C at $x = 2$.

looking for $y = mx + b$

m is $\frac{dy}{dx}|_{(2, ?)}$

$$\frac{d}{dx}((x-1)^2 + y^2) = \frac{d}{dx}(9)$$

note if $x=2$

$$2(x-1) + 2y \frac{dy}{dx} = 0$$

$$\text{then } (2-1)^2 + y^2 = 9$$

$$\frac{dy}{dx} = -\frac{2(x-1)}{2y} = \frac{-x+1}{y}$$

$$\Rightarrow y^2 = 9 - 1 = 8$$

$$\Rightarrow y = \pm\sqrt{8}$$

there are 2 lines tang.

$$m \text{ is } \frac{-2(2-1)}{2\sqrt{8}} \text{ or } \frac{-(2-1)}{\sqrt{8}}$$

$$\text{so } -\frac{3}{\sqrt{8}} \text{ or } \frac{3}{\sqrt{8}}$$

$$\sqrt{8} = \frac{-3}{\sqrt{8}}(2) + b \quad \left\{ -\sqrt{8} = \frac{3}{\sqrt{8}}(2) + b \right.$$

$$b = \sqrt{8} + \frac{6}{\sqrt{8}} \quad \left\{ b = -\sqrt{8} - \frac{6}{\sqrt{8}} \right.$$

$$y = \frac{-3}{\sqrt{8}}x + \sqrt{8} + \frac{6}{\sqrt{8}} \quad \left\{ y = \frac{3}{\sqrt{8}}x - \sqrt{8} - \frac{6}{\sqrt{8}} \right.$$

- (c) Find the point that the above line crosses the x -axis.

i.e. what x value makes $y=0$

$$\text{so } 0 = \frac{-3}{\sqrt{8}}x + \sqrt{8} + \frac{6}{\sqrt{8}}$$

$$\Rightarrow -\sqrt{8} - \frac{6}{\sqrt{8}} = \frac{-3}{\sqrt{8}}x \quad \left\{ x = \frac{8}{3} + 2 = \frac{14}{3} \right.$$

3. A ladder is 10 feet long and leaning against a wall with its base x feet away from the base of the wall.

- (a) Draw a picture of the situation described above and label y as the vertical distance from the tip of the ladder to the floor.

- (b) Find a relationship between x and y .

$$x^2 + y^2 = 10^2$$

$$x^2 + y^2 = 100$$



- (c) Find the rate that the vertical distance is moving as you change the horizontal distance.

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(100)$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$