

Exam 2

Key

TMath 124

Winter 2010

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function and x and y be positive numbers.

T F $\sqrt{4+x} = 2 + \sqrt{x}$

let x be 1

T F $\log(y) = \frac{1}{y}y'$

meant $\frac{d}{dx}(\log y) = \frac{1}{y}y'$

T F $\frac{d}{dt}(s^2) = 2s$

$\frac{d}{dt}(s^2) = 2s \frac{ds}{dt}$

T F $\frac{d}{dx}(2^x) = x2^{x-1}$

can't use power rule

(T) F $\frac{d}{dx}(\log_2(x)) = \frac{1}{x \ln 2}$

(I was assuming this equality holds for all x in the domain).

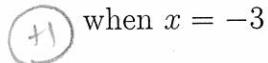
T F If f is a continuous function, f' exists.

see #2

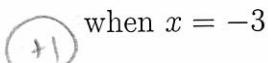
Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [5] Sketch the graph of an example function f that satisfies the following conditions:

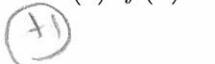
- (a) f is not differentiable when $x = -3$



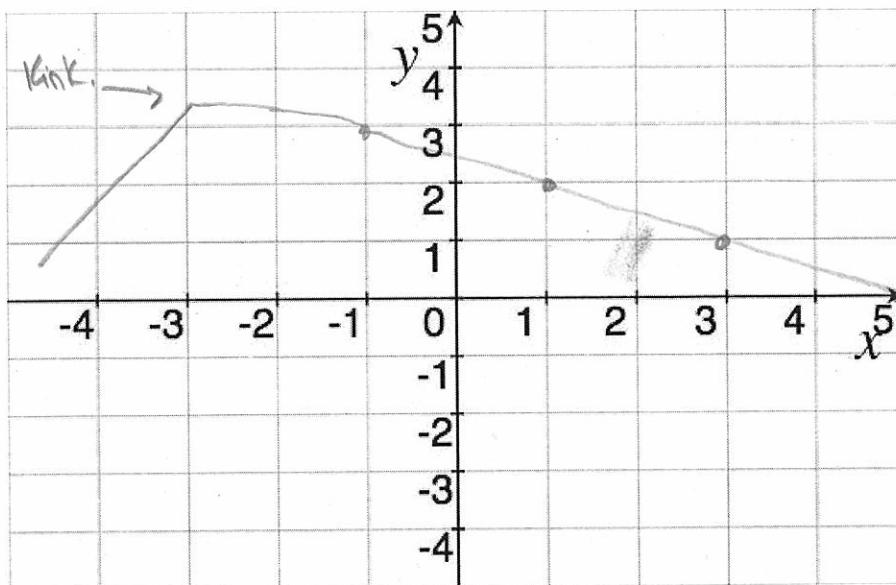
- (b) f is continuous when $x = -3$



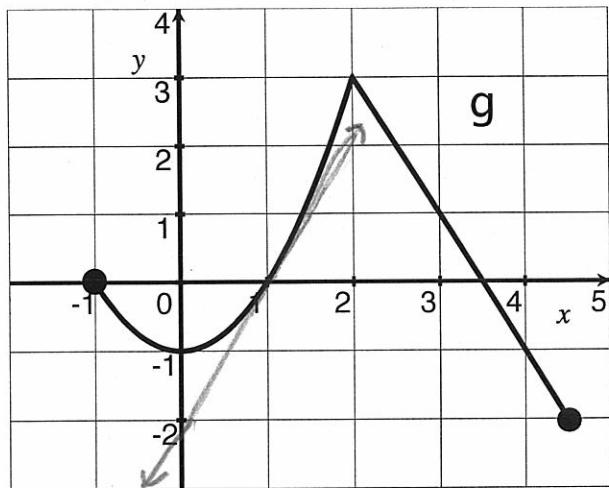
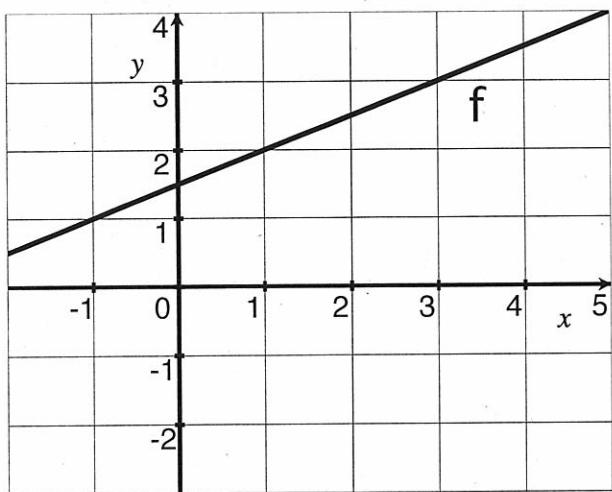
(c) $f(1) = 2$



(d) $f'(1) = -\frac{1}{2}$



3. [10] Let the graph of f and g be those shown below.



Estimate the following (if they exist):

$$(\text{WebHW8 #1}) (f - 4g)'(3)$$

$$(\text{WebHW7 #9}) f'(3) - 4g'(3)$$

$$\begin{aligned} & \frac{1}{2} - 4(-2) \\ & \frac{1}{2} + 8 = 8.5 \end{aligned}$$

$$(g + f)'(2)$$

$$g'(2) + f'(2)$$

not defined

dist over addition $\frac{1}{2}$

not def $\frac{1}{2}$

$$(f \circ g)'(1)$$

chain $\frac{1}{2}$
correct chain $\frac{1}{2}$
notation $\frac{1}{2}$

$$f'(g(1))g'(1)$$

$$\begin{aligned} & f'(5)g'(1) \\ & (\frac{1}{2})(2) \end{aligned}$$

|

$$(f \cdot g)'(3)$$

Product $\frac{1}{2}$
correct product $\frac{1}{2}$

$$f'(3)g(3) + f(3)g'(3)$$

$$(\frac{1}{2})(1) + (3)(-2)$$

$$\frac{1}{2} - 6$$

$$-5.5$$

4. [14] Find the derivatives of the following: *Do not simplify*

$$y = \sin(x) \cos(x)$$

$$[\sin(x)]' \cos(x) + \sin(x)[\cos(x)]'$$

+S

$$\cos x \cos x + \sin x (-\sin x)$$

$$(\cos x)^2 - \sin^2 x$$

$$\cos^2 x - \sin^2 x$$

product *+S*
right product *+S*
notation *+S*
got it *+S*

$$f(x) = \sin x \quad f'(x) = \cos x$$

$$g(x) = \cos x \quad g'(x) = -\sin x$$

$$f'(g(x)) g'(x)$$

$$f'(\cos x) \cdot (-\sin x)$$

$$\cos(\cos x) \cdot -\sin x$$

chain *+S*
correct chain *+S*
notation *+S*

$$y = \frac{x\sqrt{x^4+2}}{(4x-3)^7}$$

introduce ln *+S*
log prop *+S*

$$\ln y = \ln \left(\frac{x(x^4+2)^{\frac{1}{2}}}{(4x-3)^7} \right)$$

$$\ln y = \ln x + \frac{1}{2} \ln(x^4+2) - 7 \ln(4x-3)$$

$$\frac{1}{y} y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x^4+2} \cdot 4x^3 - 7 \cdot \frac{1}{4x-3} \cdot 4$$

$$y' = y \left(\frac{1}{x} + \frac{4x^3}{2(x^4+2)} - \frac{28}{4x-3} \right)$$

$$y = (\sqrt{x})^x$$

+S introduced ln
ln prop *+S* alg prop *+S*

$$\ln y = \ln(x^{\frac{1}{2}})^x$$

$$\ln y = \ln x^{\frac{1}{2}x}$$

$$\ln y = \left(\frac{1}{2}x\right) \ln x$$

$$\frac{1}{y} y' = \frac{1}{2}x \cdot (\ln x)' + \left(\frac{1}{2}x\right)' \ln x$$

$$\frac{1}{y} y' = \frac{1}{2}x \cdot \frac{1}{x} + \frac{1}{2} \ln x$$

$$\frac{1}{y} y' = \frac{1}{2} + \frac{1}{2} \ln x$$

$$y' = y \left(\frac{1}{2} + \frac{1}{2} \ln x \right)$$

+S solved by

$$\sqrt{x} \left(\frac{1}{2} + \ln \sqrt{x} \right)$$

$$y = \frac{(4x-3)^7 [\sqrt{x^4+2}]' - x\sqrt{x^4+2} [(4x-3)^7]'}{(4x-3)^7 [\sqrt{x^4+2}]^2}$$

product *+S*
right *+S*

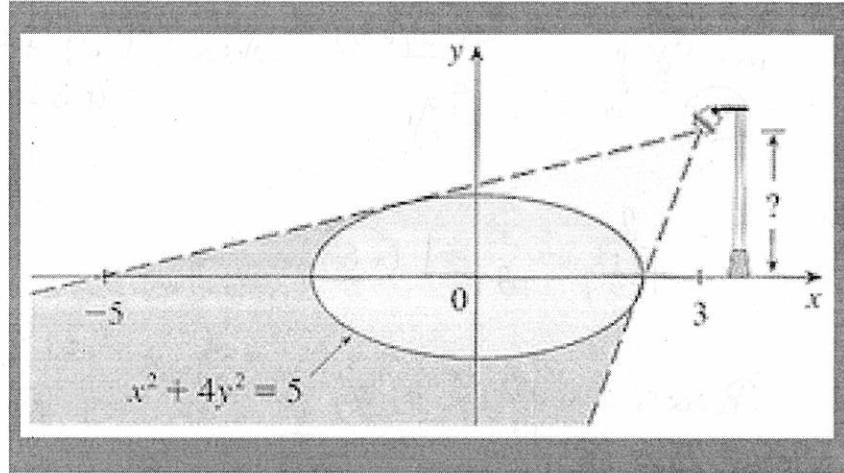
chain *+S*
right *+S*

$$= (4x-3)^7 \left[x(\sqrt{x^4+2})' + [\sqrt{x^4+2}] \right] - x\sqrt{x^4+2} 7(4x-3)^6 \cdot 4$$

or
expected order *+S*

$$= (4x-3)^7 \left(x \cdot \frac{1}{2}(x^4+2)^{-\frac{1}{2}} \cdot 4x^3 + \sqrt{x^4+2} \right) - 28x\sqrt{x^4+2}(4x-3)^6$$

5. [10] (§3.6 #69) The figure below shows a lamp located three units to the right of the y -axis and a shadow created by the elliptical region $x^2 + 4y^2 \leq 5$. The point $(-5, 0)$ is on the edge of the shadow.



(a) [2] Find $\frac{dy}{dx}$ of the ellipse

$$x^2 + 4y^2 = 5$$

$$8yy' = -2x$$

$$y' = \frac{-2x}{8y} = \frac{-x}{4y} \quad \text{solved for } y' \quad (+5)$$

(b) [3] Denote the point that is both on the ellipse and the top dashed line by (c, d) .

Notice that the slope of the top dashed line is thus $\frac{d-0}{c-(-5)}$. Use this information and what you found in part (a) to find the value of c .

$$\frac{d-0}{c+5} = m \quad \text{but } m = \frac{-c}{4d} \quad \text{thus}$$

$$\frac{d-0}{c+5} = \frac{-c}{4d}$$

ptm cstds. (+5)

$$\Rightarrow \frac{d}{c+5} = \frac{-c}{4d} \quad \text{alg (+5)} \quad \Rightarrow 4d^2 = -c(c+5) \quad \text{sub in } d \quad (+5) \quad b/c \quad (c, d) \text{ is on the ellipse}$$

$$4d^2 = 5c^2 \quad \Rightarrow \quad 5c^2 = -c^2 - 5c \quad \Rightarrow \quad 5 = -5c \quad \Rightarrow \quad c = -1$$

(c) [5] Find the equation of the top dashed line and then find out the height of the lamp. AH: [4] Find the eq of line tangent to $x^2 + 4y^2 = 5$ when $x = -2$ and $y > 0$.

The x -coord is -1 so the y -coord is $(-1)^2 + 4y^2 = 5$

$$\Rightarrow 4y^2 = 4 \quad \Rightarrow \quad y^2 = 1 \quad \Rightarrow \quad y = \pm 1 \quad b/c \quad \text{above } x\text{-axis, } y = 1. \quad (+5) \quad \text{find } y \text{ when } x = -1$$

$$\therefore \text{slope is } -\frac{1}{4} \quad (+1) \quad \text{Sad (+5)}$$

$$\text{line: } y = \frac{1}{4}x + b \quad \text{looking for line (+5)}$$

passes thro $(-1, 1)$ so

$$1 = \frac{1}{4}(-1) + b \quad \Rightarrow \quad b = 1.25 \quad (+5)$$

\Rightarrow height is

$$\frac{1}{4}(3) + 1.25 = \frac{3}{4} + 1.25 = 2 \text{ units}$$

plug in 3 to correct eq (+5)

6. (§3.9 #21) [5] Ryan and Stella were being chased by a pack of zombies. At point P they decided to split up and Stella ran south at 12 ft/s. Ryan waited for three minutes to try to draw most of the zombies towards him and then started to run east at 15 ft/s. Ten minutes later the two of them are still alive and running in their respective directions. At what rate are Ryan and Stella moving apart at this moment in time?

