

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let  $f$  be a function and  $x$  and  $y$  be positive numbers.

T  F  $(x+y)^{-2} = \sqrt{x+y}$

$$(x+y)^{-2} = \frac{1}{(x+y)^2}$$

T F  $\lim_{x \rightarrow a} [3f(x)] = 3 \lim_{x \rightarrow a} f(x)$

T  F  $\lim_{x \rightarrow a} f(x) = f(\lim_{x \rightarrow a} x)$

only if  $f$  is cont.

T F If  $f'(a)$  exists, then  $\lim_{x \rightarrow a} f(x) = f(a)$ . If  $f$  is diff, then  $f$  is cont

T  F The absolute value function is a differentiable function.

~~↙ corner~~

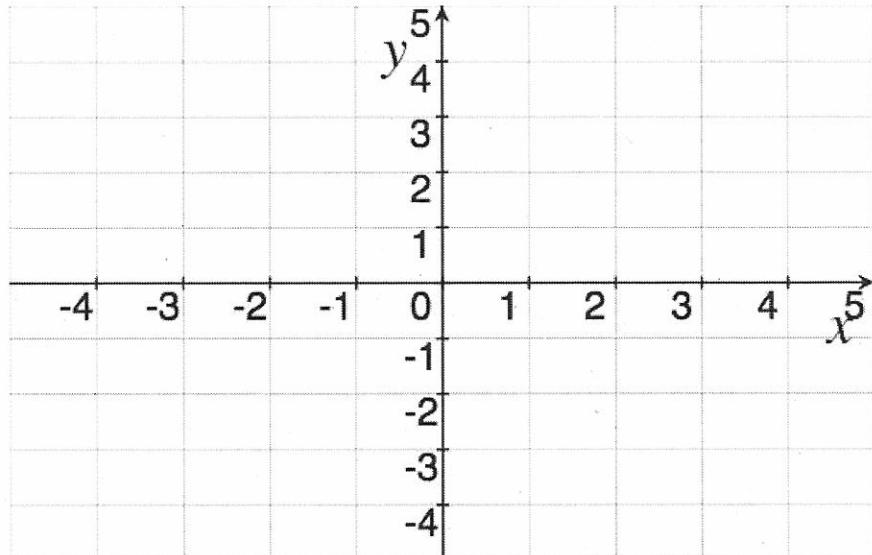
T  F  $\frac{d}{dx}(e^x) = xe^{x-1}$

$$\frac{d}{dx}(e^x) = e^x$$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (Quiz 2 #2) Sketch the graph of an example function  $f$  that satisfies the following conditions:

- (a)  $f$  is continuous everywhere but when  $x = 3$



- (b)  $\lim_{x \rightarrow 3^-} f(x) = \infty$



- (c)  $f(-1) = 2$



- (d)  $f'(-1) = 0$



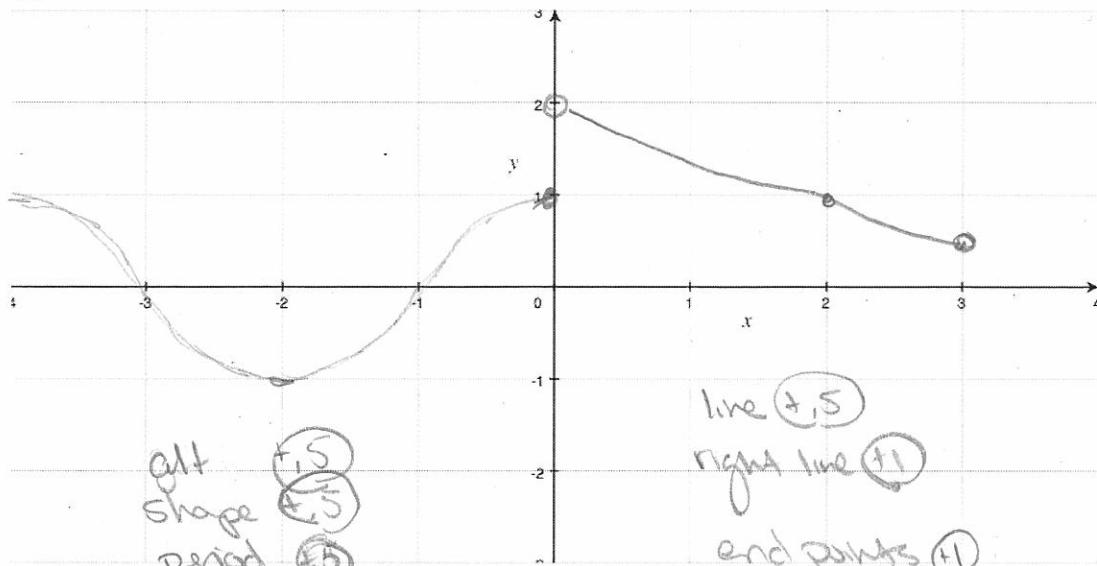
3. Let

$$g(x) = \begin{cases} \cos\left(\frac{\pi}{2}x\right) & \text{if } x \leq 0, \\ -\frac{1}{2}x + 2 & \text{if } 0 < x < 3. \end{cases}$$

$$\frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$$

+ period of 4

[3] Carefully graph  $g$  below.



[3 each] Use the graph above to find the following (if they exist!):

[1]  $\lim_{x \rightarrow 0^+} g(x)$

2

[1]  $\lim_{x \rightarrow -\infty} g(x)$

does not exist

[2]  $g'(2)$

-1/2

signe +.5  
slope +1  
got it +.5

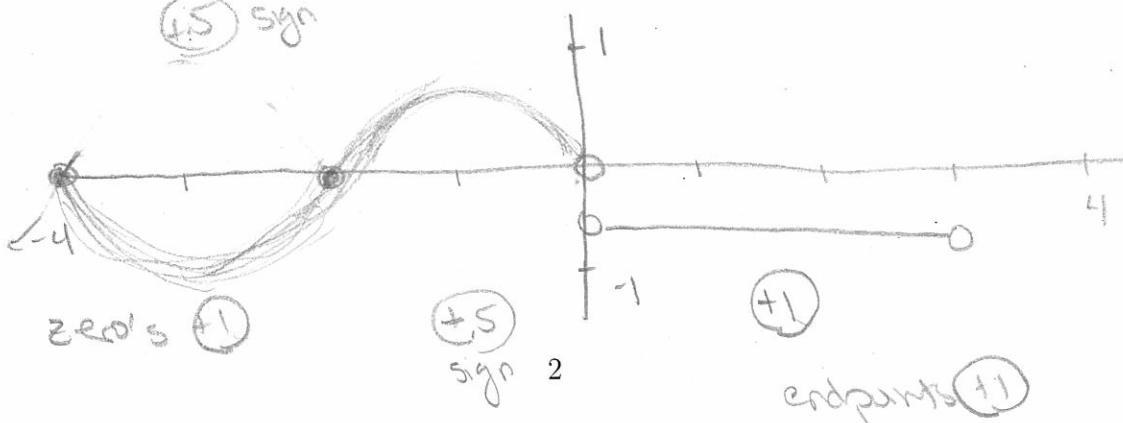
[2]  $\lim_{x \rightarrow -2} [6g(x) - 5]$

6(-1) - 5

-6 - 5 = -11

[4] (§2.8 #9) Make a rough sketch of the graph of  $g'(x)$ :

(+) sign



4. [each] (§2.3 #3 & CalcWebW5 #4) Find the limit or explain why it does not exist.

$$\lim_{x \rightarrow -1} (3x^4 + 2x^2 - x + 1)$$

b/c polynomials are cont.

$$3(-1)^4 + 2(-1)^2 - (-1) + 1$$

$$3 + 2 + 1 + 1 = 7$$

notation (1)

stated (1)

plug in (1)

alg (1)

$$\lim_{x \rightarrow \infty} \frac{4 - 7x^2}{8x^2 + 3x}$$

~~top~~  $\cancel{x^2}$  took (1.5)  
~~bottom~~  $\cancel{x^2}$  right one (1)

$$= \lim_{x \rightarrow \infty} \frac{4\cancel{x^2} - 7}{8 - 3\cancel{x}}$$

$$= -7/8$$

notation (1.5)

took limits (1)

alg (1)

$$\lim_{h \rightarrow 0} \frac{(2+h)^2 - (2+h) - 2}{h}$$

stated (1.5)

notation (1)

alg (1.5)

limit laws (1)

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x^2}$$

note

$$= \lim_{h \rightarrow 0} \frac{4 + 4h + h^2 - 2 - h - 2}{h}$$

$$(1) -1 \leq \sin x \leq 1$$

$$= \lim_{h \rightarrow 0} \frac{3h + h^2}{h}$$

$$\Rightarrow b/c \frac{1}{x^2} is positive$$

$$= \lim_{h \rightarrow 0} \frac{h(3+h)}{h}$$

$$-\frac{1}{x^2} \leq \frac{1}{x^2} \sin x \leq \frac{1}{x^2} \text{ took limit (1)}$$

$$= \lim_{h \rightarrow 0} 3h = 3 + 0 = 3$$

$$b/c \lim_{x \rightarrow \infty} -\frac{1}{x^2} = 0$$

$$\& \lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$$

(1) the squeeze thm

$$\Rightarrow \lim_{x \rightarrow \infty} x^2 \sin x = 0$$

explain/notation (1.5)

5. [3] (CalcWebHW5 #13) If the tangent line to  $y = f(x)$  at  $(4, 3)$  passes through the point  $(0, 2)$  find the following.

(a)  $f(4) = 3$  +1

(b)  $f'(4) = \text{slope of line tangent to graph at } (4, 3)$  +1

$$= \frac{3-2}{4-0} = \frac{1}{4}$$

+1

6. Let  $f(x) = x^2 - e$ , where  $e$  is approximately 2.718.

- (a) [4] (Nice Derivative Wks #3) Find the equation for the line tangent to the graph of  $f$ , when  $x = 1$ .

looking for  $y = mx + b$ . +1

$$\begin{aligned} m &= \text{slope of line tangent to } f \text{ when } x = 1 \\ &= f'(1). \quad \text{+1} \end{aligned}$$

$$\text{note } f'(x) = \frac{d}{dx}(x^2 - e) = 2x \quad \text{+1}$$

$$\text{so } f'(1) = 2 \cdot 1 = 2 \quad \text{+1}$$

- (b) [3] (Nice Derivative Wks #4) Find the point on the graph of  $f$  whose tangent line is parallel to the line defined by  $3y = x + 3$ .

We want to find  $x$  so that

$$\begin{aligned} \text{the slope of the} \\ \text{line tangent to } f \text{ at } x &= \frac{1}{3} \quad \text{+1} \end{aligned}$$

$$\frac{d}{dx}(x^2 - e) = \frac{1}{3} \quad \text{+1}$$

$$\text{derivative} \quad 2x = \frac{1}{3}$$

$$x = \frac{1}{6} \quad \text{+1}$$

we have  $y = 2x + b$   
Note the line passes thru  
 $\textcircled{+1} (1, f(1)) = (1, -1.718)$

so

$$\begin{aligned} -1.718 &= 2(1) + b \\ \Rightarrow b &= -3.718 \end{aligned}$$

$$\text{so } y = 2x - 3.718 \quad \text{+1}$$

$$3y = x + 3$$

$$y = \frac{1}{3}x + 1$$

so when  $x = y_6$