EXAM 2 # TMath 124 Spring 2023

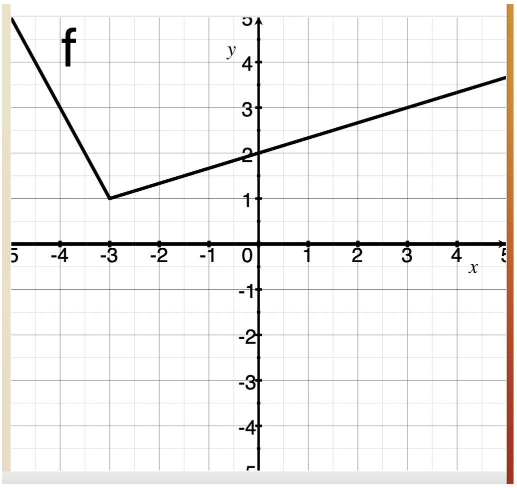
Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

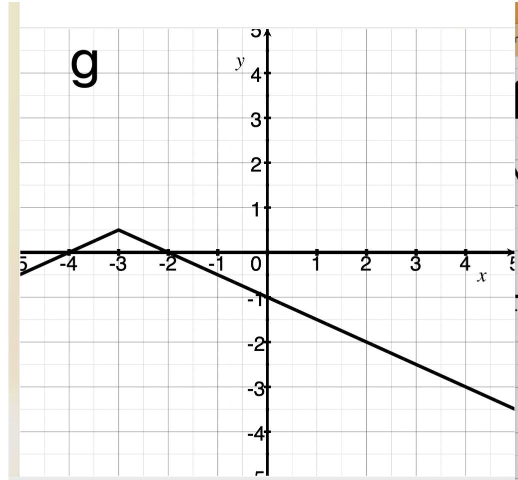
1. [2] (TrigAcitvity#2) Find $\lim_{x \to 0} \dfrac{\sin(2x)}{6x}$

2. Identify which derivative rule(s) you can use to find $\frac{dy}{dx}$. Do *not* find $\frac{dy}{dx}$!!

	Derivative Rule(s)
[2] (ImpExpActivity#5) $y = \sqrt{\frac{x-1}{x^4+1}}$	
[2] (§3.5 #24) $y + x\ln(y) = x^9$	
[2] (WebHW8 #9) $y = e^{x^3-5x}$	
[2] (WebHW10 #13) $y = (\tan(x))^x$	

3. Use the graphs of f and g below for the following questions.





(a) [2] (ProductActivity#1) Find an x so that $g'(x)$ does not exist.

(b) [3] (WebHW8#7) Estimate $\frac{d}{dx}(f(x)g(x))|_{x=0}$

(c) [3] (Quiz3#1) If $c(x) = f(g(x))$, then estimate $c'(4)$.

(d) [3] (§3.4#72) If $h(x) = g(3x - 1)$, then estimate $h'(2)$.

4. The differentiable functions f and g are defined for all real numbers. Values for f, f', g, and g' for various x values are given in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	2	6
2	1	5	8	7
3	7	7	2	9

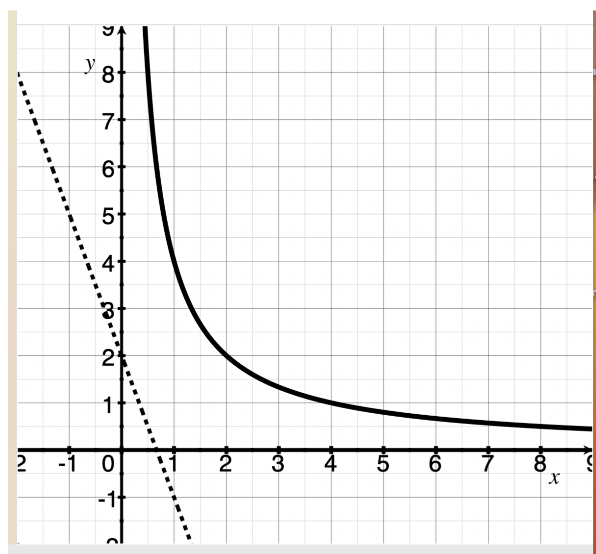
(a) [4] (PracticeExam#4) Given that $h(x) = \frac{f(x)}{3x+g(x)}$, find $h'(1)$.

(b) [3] (§3.10 #52) Find the linearization of f at $x = 2$.

(c) [2] (§3.10 #52) Use the linearization of f to approximate $f(2.05)$.

5. A particle moves along a hyperbola $xy = 4$ when $x > 0$. The graph is shown below with a solid curve. The dotted line is of a dust particle moving along a straight line.

(a) [4] (§3.2 #56) Find the point that the particle's movement is parallel to a dust particle moving along the dotted straight line graphed.



(b) [4] (WebHW11#5) When the particle reaches an x value of 1, the y-coordinate is decreasing at a rate of 3 cm/s. How fast is the x-coordinate of the point changing at that instant?

6. (RelatedActivity#2) On April 20, 2010 the Deepwater Horizon drilling rig exploded initiating the worst oil spill in US history. It was important to estimate the volume of oil spewing out each day, but it is hard to measure such a high volume flow directly. Instead one can measure the area of the spill from the air and the thickness of the spill and compute backwards. Suppose that the spilled oil is cylindrical in shape and a uniform thickness of 0.001m. On day 9 of the spill the area of the spill was 13,000,000 m^2 and the radius of the spill was increasing at a rate of 10 meters per day (Modified from Dr. Dawn's Blog)

   (a) [3] Find a formula for how fast the volume is changing t days after the explosion.

   (b) [2] At what rate was the volume of the spill increasing on the 9th day?

   (c) [1] BP's original/official estimates of the flow rate were 160 to 790 m^3/day, how accurate were their estimates?