EXAM 2 # TMath 124 Spring 2023

Key

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. [2] (TrigAcitvity#2) Find $\lim_{x\to 0} \frac{\sin(4x)}{6x}$

algebraically: (+.5)

$\lim_{x\to 0} \frac{\sin(4x)}{6x} \cdot \frac{4}{4} = \lim_{x\to 0} \frac{4}{6} \cdot \frac{\sin(4x)}{4x}$ (+.5) $\to 1$ (+.5)

$= 4/6 = 2/3$ got it (+.5)

OR

Graphically: (+.5)

$(0, 2/3)$

shape (+1)

so $2/3$ got it (+.5)

OR

Numerically: (+.5) both sides (+.5)

x	$-.01$	$-.001$	$.001$
$\frac{\sin(4x)}{6x}$	$.6664$	$.\overline{66}$	$.\overline{66}$

eval (+.5)

so $2/3$ got it (+.5)

2. Identify which derivative rule(s) you can use to find $\frac{dy}{dx}$. Do *not* find $\frac{dy}{dx}$!!

	Derivative Rule(s)
[2] (ImpExpActivity#5) $y = \sqrt{\frac{x-1}{x^4+1}}$ Start (+.5)	logarithmic differentiation (+1) (ln both sides) (use ln properties) (implicitly differentiate) and chain rule 2 times (+.5) OR Chain rule (+1) outside: $u^{1/2}$ inside: $\to$ need quotient rule (+.5)
[2] (§3.5 #24) $y + x4^y = x^9$ Start (+.5)	differentiate each term use product rule on $x4^y$ (+.5) and implicit differentiation (+1) with power rule in places OR algebra for $x4^y = x^9 - y$ (+1) logarithmic differentiation (to break up product) then chain rule 3 times
[2] (WebHW8 #9) $y = e^{x^3-5x}$ Start (+.5)	Chain Rule (+1) outside: e^u inside: x^3-5x $\to$ power rule (+.5) OR logarithmic differentiation (+1) (ln both sides) (use ln properties) (implicitly differentiate) with power rule in places (+.5)
[2] (WebHW10 #13) $y = (\tan(x))^x$ Start (+.5)	logarithmic differentiation (+1) (ln both sides) use ln properties to pull down x in exp (implicitly differentiate) with product rule (+.5)

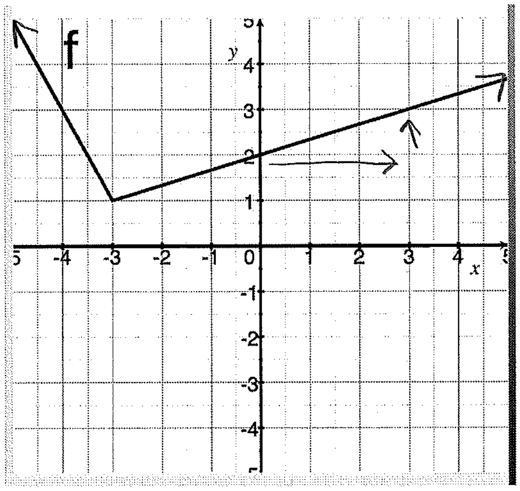
note: we cannot use power rule b/c x is an exponent

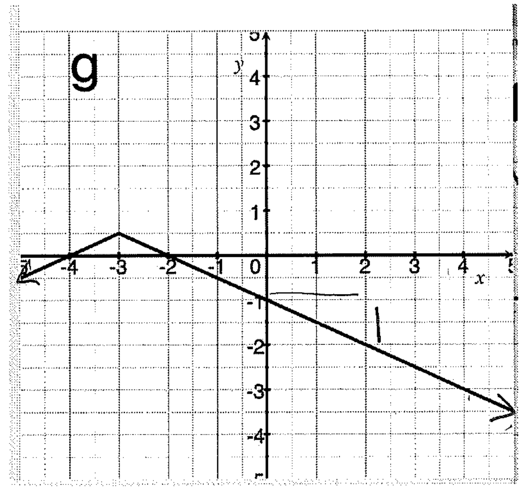
note: we cannot use $\frac{d}{dx}(b^x)$ b/c base depends on x

$\Rightarrow$ we have to use logarithmic differentiation

10

3. Use the graphs of f and g below for the following questions.





(a) [2] (ProductActivity#1) Find an x so that $g'(x)$ does not exist.

$x = -3$ note the corner when $x = -3$

(b) [3] (WebHW8#7) Estimate $\frac{d}{dx}(f(x)g(x))|_{x=0}$

$\frac{d}{dx}(f(x)g(x))\Big|_{x=0} = f(0)\,\frac{dg}{dx}\Big|_{x=0} + \frac{df}{dx}\Big|_{x=0}\cdot g(0)$ product rule (+.5) use right (+.5)

$= 2 \cdot \frac{-1}{2} + \frac{1}{3}(-1) = -1 - \frac{1}{3} = \frac{-4}{3}$

(+.5) (+.5) (+.5) (+.5)

(c) [3] (Quiz3#1) If $c(x) = f(g(x))$, then estimate $c'(4)$.

$c'(4) = f'(g(4)) \cdot g'(4)$ chain rule (+.5) use right (+.5)

$= f'(-3) \cdot \left(-\frac{1}{2}\right)$ notation (+.5)

(+.5) (+.5)

(+.5) DNE?? corner when $x = -3$?

(d) [3] (§3.4#72) If $h(x) = g(3x-1)$, then estimate $h'(2)$.

$h'(2) = g'(3(2)-1) \cdot \frac{d}{dx}(3x-1)\Big|_{x=2}$ chain rule (+.5) use right (+.5)

$= g'(5) \cdot 3$ Composition (+.5)

(+.5) (+.5)

$= -\frac{1}{2} \cdot 3 = \frac{-3}{2}$

(+.5)

4. The differentiable functions f and g are defined for all real numbers. Values for f, f', g, and g' for various x values are given in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	2	6
2	1	5	8	7
3	7	7	2	9

(a) [4] (PracticeExam#4) Given that $h(x) = \frac{f(x)}{2x+g(x)}$, find $h'(1)$. (+.5)

quotient (+.5)
did right (+.5)
notation (+.5)

$h'(1) = \dfrac{(2(1)+g(1))\cdot f'(1) - f(1)[2+g'(1)]}{[2(1)+g(1)]^2}$

evaluating (+2) $\dfrac{(2+2)4 - 3(2+6)}{(2+2)^2} = \dfrac{16-3(8)}{4^2} = \dfrac{16-24}{16} = \dfrac{-8}{16} = -\dfrac{1}{2}$

(b) [3] (§3.10 #52) Find the linearization of f at $x = 2$.

notation (+.5)

Looking for $y - y_1 = m(x - x_1)$ (+.5)
m = slope of line tangent to f @ $x=2$
$= f'(2) = 5$ (+.5)

So (+.5) $y - 1 = 5(x-2)$ (+.5) plug in (+.5)
or
$y = 5x - 9$

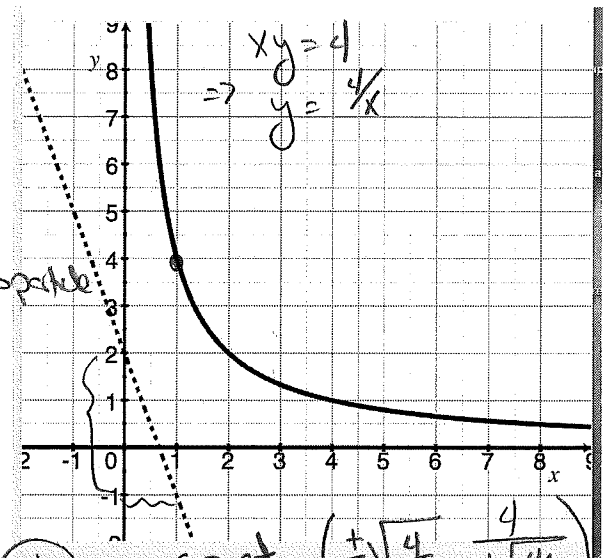
(c) [2] (§3.10 #52) Use the linearization of f to approximate $f(2.05)$.

notation (+.5)

ie plug 2.05 into the line found in (b) (+1)

$5(2.05) - 9 = 1.25$ (+.5)

5. A particle moves along a hyperbola $xy = 4$ when $x > 0$. The graph is shown below with a solid curve. The dotted line is of a dust particle moving along a straight line.



(a) [4] (§3.2 #56) Find the point that the particle's movement is parallel to a dust particle moving along the dotted straight line graphed.

ie find x when
(+.5) slope of dotted line = slope at tang. line to particle
$\frac{-3}{1} = y'(x)$ (+.5) (+.5)
note $y = 4/x$ (+.5) $\Rightarrow y' = -4x^{-2}$ (+1)
$\Rightarrow -3 = -4x^{-2}$
$\Rightarrow -3 = \frac{-4}{x^2}$
$\frac{-3x^2}{-3} = \frac{-4}{-3}$
$x = \pm\sqrt{\frac{4}{3}}$ alg (+1)
so at $\left(\pm\sqrt{\frac{4}{3}}, \frac{4}{\pm\sqrt{4/3}}\right)$

(b) [4] (WebHW11#5) When the particle reaches an x value of 1, the y-coordinate is decreasing at a rate of 3 cm/s. How fast is the x-coordinate of the point changing at that instant?

(+.5) $\left.\frac{dy}{dt}\right|_{x=1} = -3$ cm/s WANT $\left.\frac{dx}{dt}\right|_{x=1}$ (+.5)

$\frac{d}{dt}$ (+.5) $xy = 4$
(+1) $x\frac{dy}{dt} + \frac{dx}{dt}y = 0$

when $x = 1$: $1 \cdot (-3) + \left.\frac{dx}{dt}\right|_{x=1} y|_{x=1} = 0$ (+.5) (+.5) (+.5)
from graph $y = 4$
$\Rightarrow -3 + \left.\frac{dx}{dt}\right|_{x=1} \cdot 4 = 0 \Rightarrow \left.\frac{dx}{dt}\right|_{x=1} = \frac{3}{4}$

17

21
24
45

total points available (45)

6. (WordProblems#9) If C is the cost (\$ out) a company incurs by producing x units of their commodity, the marginal cost MC is equal to $\lim_{\Delta x \to 0} \frac{\Delta C}{\Delta x} = \frac{dC}{dx}$. Similarly, if R is the revenue (\$ in) a company gathers by producing x units on their commodity, the marginal revenue MR is equal to $\frac{dR}{dx}$. Also if P is profit (\$), the marginal profit, MP, is $\frac{dP}{dx}$. Note that Profit=(\$ in)-(\$ out)=Revenue-Cost.

   (a) [2] A company sells each product for \$450 dollars. Write down the revenue function for the company selling x units.

   (b) [3] The same company has a cost function of $C(x) = 10,000 + 3x^2$. Find the number x units that should be produced to maximize profit.

   (c) [2] Explain why ~~economies~~ economists care when $MP = 0$.

(+1) Start (+.5) notation (+.5)

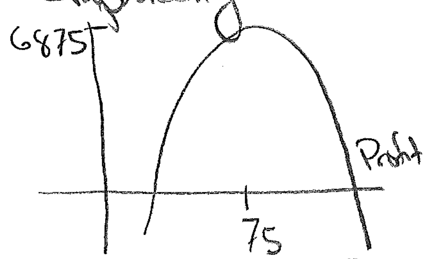
a) Revenue = \$ in = (sale price)(quantity sold) = 450x

b) Profit = \$ in − \$ out
= Revenue − Cost (+.5)
= 450x − (10,000 + 3x²) (+.5)

negative effects whole cost (+.5)

(+1.5)

Graphically:



max @ x = 75

OR Precalculus:
parabola has vertex @ $\frac{-b}{2a}$
$\frac{-450}{2(-3)} = \frac{450}{6} = 75$

OR Calculus:
when $\frac{d}{dx}(\text{Profit}) = 0$
$P'(x) = 450 - 6x$
$0 = 450 - 6x$
⇓
$x = 75$

c) Explain why economicists care when MP=0

start (+.5)

Recall MR = the derivative of the profit function or P'(x)

Note that MP=0 meaning P'(x)=0.

(+1) In this case P'(x) equals zero when the profit function maxes out - which is exactly what we wanted to do to profit P

sense (+.5)

7