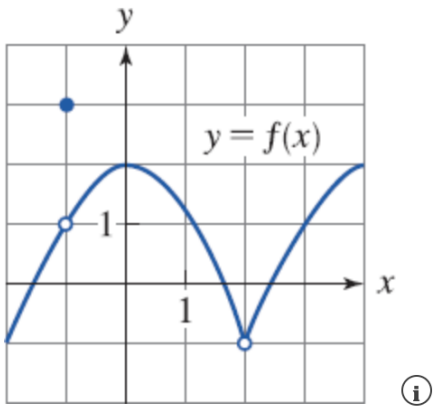
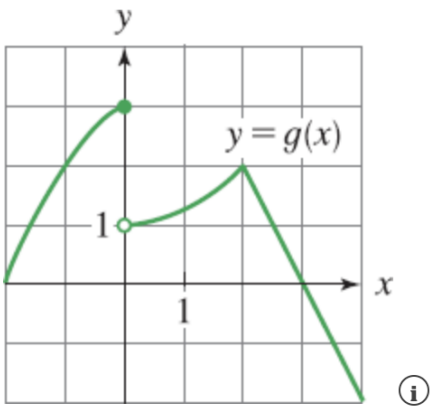
EXAM 1 # TMath 124 Spring 2023

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. The graphs of f and g are given. Use them to estimate the following:





   (a) [3] (WebHW3#2) $\lim_{x\to 2}(3f(x) - g(x))$

   (b) [3] (§2.3#2) $f(-1) + \lim_{x\to -1}(xg(x))$

   (c) [2] (Quiz2#1) $g'(3)$

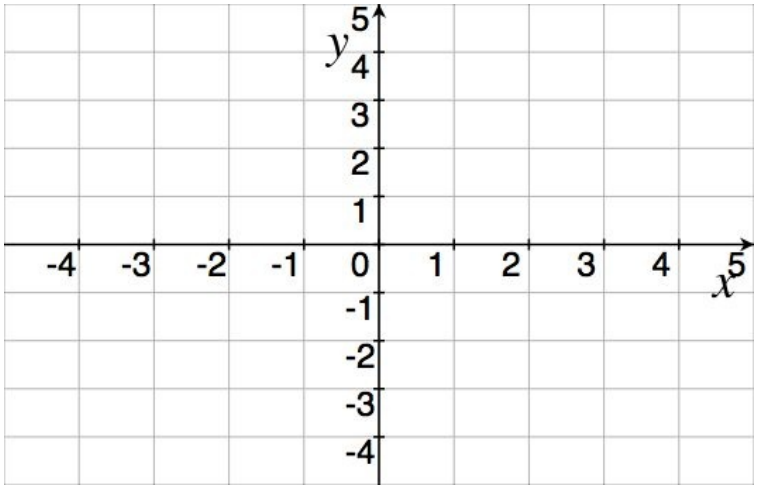
2. [5] (Quiz1#2) Draw one graph for a function $\alpha(x)$, that satisfies all of the following:

   (a) $\lim_{x\to 3} \alpha(x) = -\infty$,

   (b) α is continuous on the interval $(-2, 2)$,

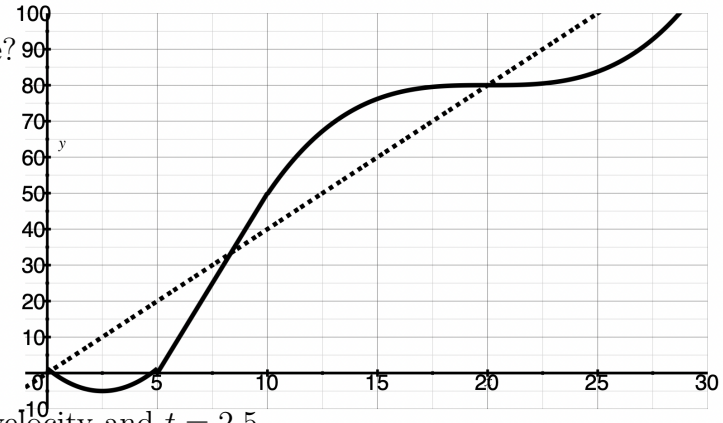
   (c) $\alpha(-2) = 1$, and

   (d) $\lim_{x\to -2^-} \alpha(x) = -1$.



3. [4] (Practice Exam#7) Let $f(x) = 4x - 3$. Find the limit (either numerically, graphically, or algebraically) if it exists of $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$

4. The solid curve, denoted R, records the distance (in meters) of Ryan from the start line after t seconds. The dotted function records the distance of Julie & is denoted J.



(a) [1] Who wins the race 100 meter race?

(b) [2] (DerivativeActivity #1) Is there a runner who moves away from the finish line? If so who and when?

(c) [2] (WebHW5 #4) Estimate Ryan's velocity and $t = 2.5$.

(d) [2] (Quiz2#1) Estimate $\frac{d}{dt}J|_{15}$.

(e) [3] (§2.7 #16) Do the runners ever have the same velocity? If so, when?

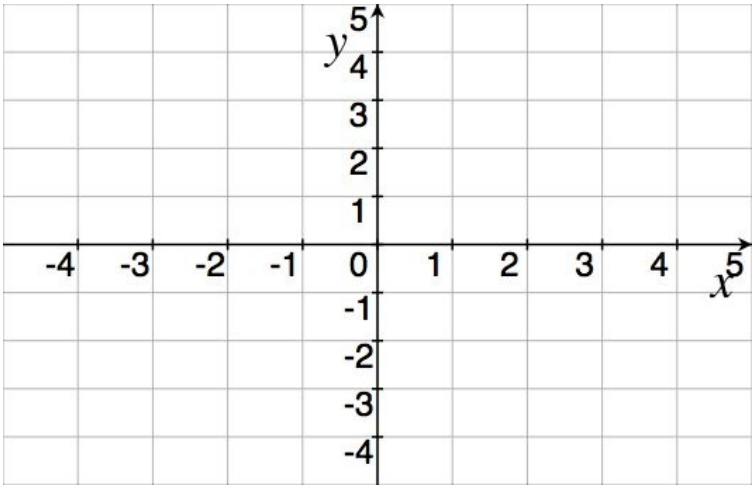
5. [5] (WebHW5#8) Draw one graph for a function $\beta(x)$, that satisfies all of the following:

   (a) $\lim_{x\to-\infty} \beta(x) = 2$,

   (b) β is continuous on the interval $(-4, 4)$,

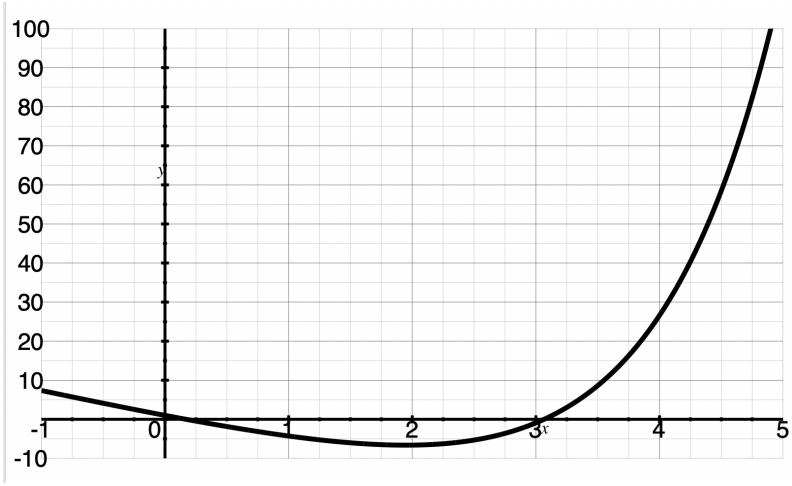
   (c) $\beta'(0)$ does not exist, and

   (d) $\frac{d}{dx}\beta'|_3 = 1$.



6. Consider $f(x) = e^x - 7x$ graphed to the right.

   (a) [3] (WebHW7#9) Find $\dfrac{df}{dx}$



   (b) [1] (DerivativeActivity#5) Estimate when $f'(x) = 0$

   (c) [3] (ExpActivity#4) Find the equation of the line tangent to f that is also horizontal.

7. (WordProblems#1) Test makers use item response functions $P(x)$ to determine the difficulty and effectiveness of a given test question. The variable x is the ability of a test taker and $P(x)$ is the probability that the test taker gets the problem correct. By convention we let an "average ability" correspond with $x = 0$. Thus $P(0) = .75$ means that a person with average ability has a 75% chance of getting the question correct.

   (a) [2] Assume we have a well constructed True/False question. Sketch a possible response function $P(x)$ so that $P(0) = .75$. Note that you do not need to put units on the x axis but should have units on the vertical axis.

   (b) [2] On a well constructed question, what do we expect $\lim_{x\to\infty} P(x)$ to equal? Justify your answer.

   (c) [2] Assume the question is a True/False question, find $\lim_{x\to-\infty} P(x)$. Justify yourself.