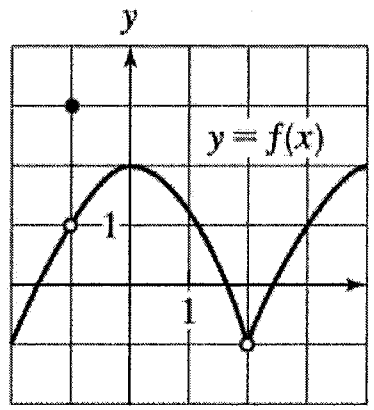
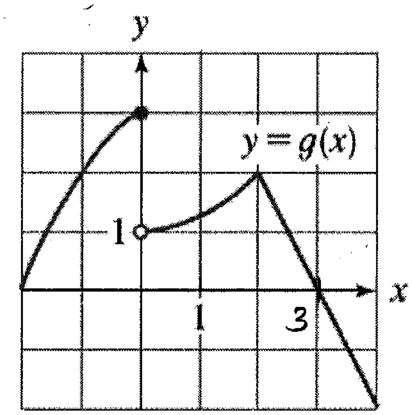
Key

EXAM 1 TMath 124 Spring 2023

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. The graphs of f and g are given. Use them to estimate the following:





(a) [3] (WebHW3#2) $\lim_{x\to 2}(3f(x) - g(x))$ $= 3\lim_{x\to 2} f(x) - \lim_{x\to 2} g(x) = 3(-1) - 2 = -5$ (+.5) (+.5) (+1) (+.5)

notation (+.5)

(b) [3] (§2.3#2) $f(-1) + \lim_{x\to -1}(xg(x))$ $= 3 + \lim_{x\to -1}(x)\lim_{x\to -1} g(x) = 3 + (-1)(2) = 1$ (+1) (+.5) (+.5) (+.5)

notation (+.5)

(c) [2] (Quiz2#1) $g'(3)$ = slope of line tangent to g at $x = 3$ $= \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$ (+.5) (+.5) (+1)

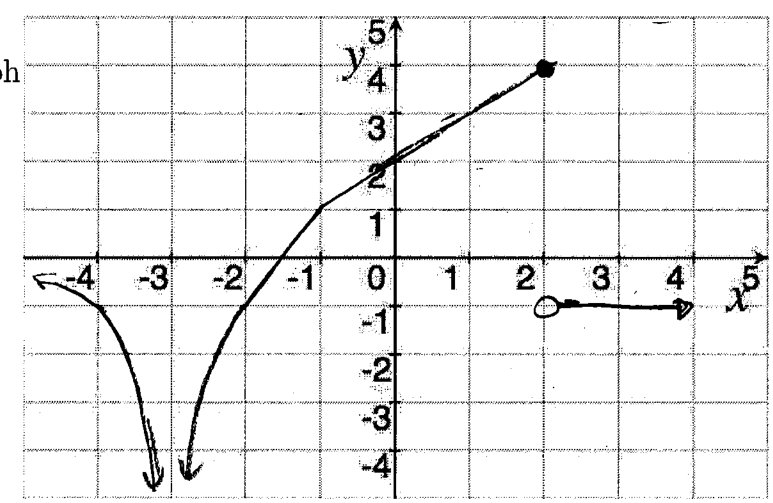
2. [5] (Quiz1#2) Draw one graph for a function (+1) $\alpha(x)$, that satisfies all of the following:

(+1) (a) $\lim_{x\to -3} \alpha(x) = -\infty$,

(+1) (b) α is continuous on the interval $(-2, 2)$,

(+1) (c) $\alpha(2) = 4$, and

(+1) (d) $\lim_{x\to 2^+} \alpha(x) = -1$.



note there are MANY correct answers!

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3. [4] (Practice Exam#7) Let $f(x) = 3x - 5$. Find the limit (either numerically, graphically, or algebraically) if it exists of $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

algebraically (+1) (+1) OR

$$\lim_{h\to 0} \frac{[3(2+h)-5]-[3\cdot 2-5]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6}+3h-\cancel{5}-\cancel{6}+\cancel{5}}{h}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 3 = 3$$

start (+.5) notation (+.5) algebra (+1)

we are computing $f'(2)$ so power rule

$\Rightarrow f'(x) = 3(1)x^0 - 0$

$\Rightarrow f'(x) = 3$

So $f'(2) = 3$

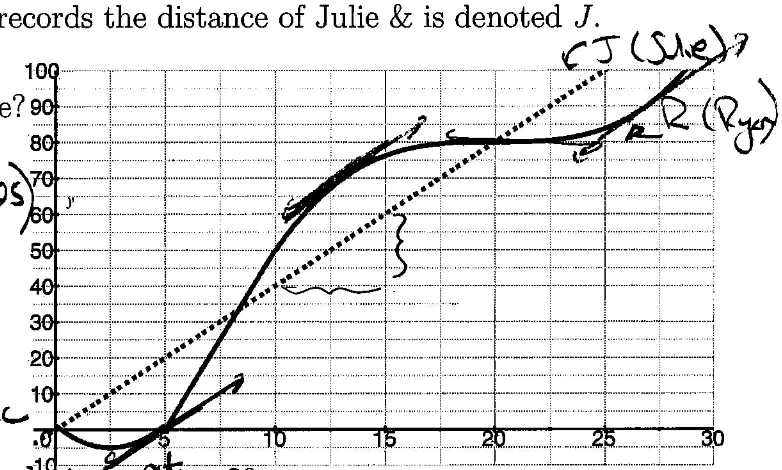
OR graphically $y=3$

so as $h \to 0$ output $\to 3$

OR numerically

h	$-.01$	$-.0001$	$.0001$	$.1$
$\frac{f(2+h)-f(2)}{h}$	4	4	4	4

4. The solid curve, denoted R, records the distance (in meters) of Ryan from the start line after t seconds. The dotted function records the distance of Julie & is denoted J.



(a) [1] Who wins the race 100 meter race?

(+.5) Julie as J reaches the y value of 100 first (at t=25)

graph reading (+.5)

(b) [2] (DerivativeActivity #1) Is there a runner who moves away from the finish line? If so who and when?

yes? Ryan does between (+.5) $0 < t < 2.5$ sec (+1)

graph reading (+.5)

(c) [2] (WebHW5 #4) Estimate Ryan's velocity and at $t = 20$.

velocity = slope of line tangent to R at $t=20$ (+1)

= slope looks horizontal = 0 (+.5)

draw line (+.5)

(d) [2] (Quiz2#1) Estimate $\frac{d}{dt}J|_{10}$.

$\frac{d}{dt}J|_{10}$ = slope of line tangent to J at $t=10$ = $\frac{20\,m}{5\,s}$ = $4\,m/s$ (+1) (+1)

(e) [3] (§2.7 #16) Do the runners ever have the same velocity? If so, when?

Yup? (+.5) Since Julie has a constant velocity of 4 m/s this boils down to finding/estimating when the lines tangent to R have a slope of 4 m/s too. (+1)

start (+.5)

(note get this bc of 1st 0 second)

This looks to happen several times?

$\approx t =$ 4 sec (+.5), 13 sec (+.5) and ~~26 sec (+.5)~~

note Julie is not running?

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5. [5] (WebHW5#8) Draw one graph for a function $\beta(x)$, that satisfies all of the following: (+1)

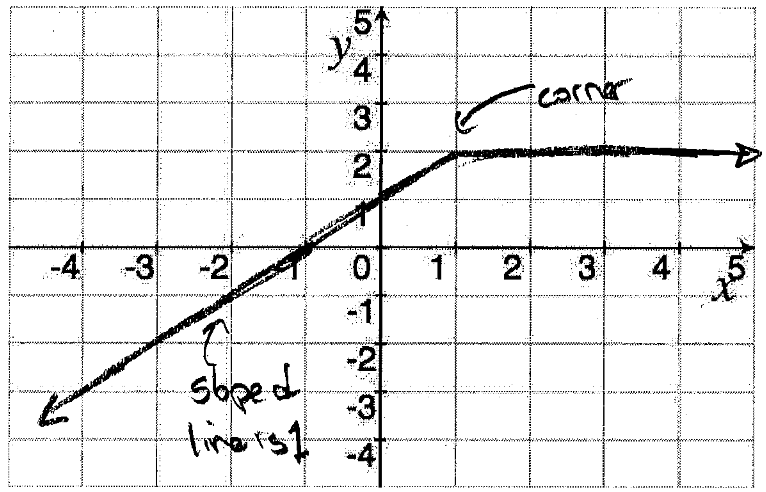
(+1) (a) $\lim_{x\to\infty} \beta(x) = 2$,

(+1) (b) β is continuous on the interval $(-4, 4)$,

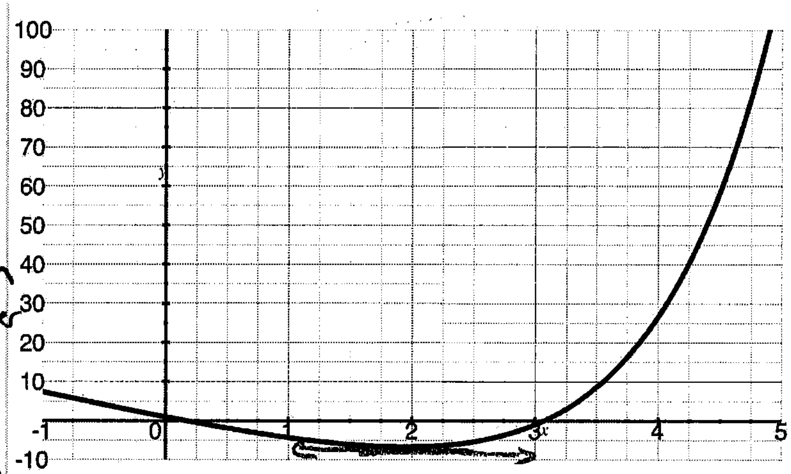
(+1) (c) $\beta'(1)$ does not exist, and

ignore? (+1) (d) $\frac{d}{dx}\beta\Big|_{-2} = 1$.

Note there are MANY correct answers.



6. Consider $f(x) = e^x - 7x$ graphed to the right.



(a) [3] (WebHW7#9) Find $\frac{df}{dx}$

power rule + exp rule:

$f'(x) = e^x - 7(1)x^0$ (+.5) notation (+.5) dist over sub

$= e^x - 7$ (+1) (+1)

or

technology/calculator/Desmos

(b) [1] (DerivativeActivity#5) Estimate when $f'(x) = 0$ ie when horiz. tangent (+.5)

$\approx x = 2$ (+.5)

(c) [3] (ExpActivity#4) Find the equation of the line tangent to f that is also horizontal. (+.5) Looking for $y = mx + b$

m = slope of line tangent to f at point

= horizontal line

$= 0$

The point on f was not given.. we need to find x value so

(+1) $f'(x)$ = horizontal line

$e^x - 7 = 0$

$\Rightarrow e^x = 7 \Rightarrow x = \ln 7 \approx 1.946$ alg (+.5)

When $x = \ln 7$, $y = e^{\ln 7} - 7(\ln 7)$

(+.5) find y-value

$= 7 - 7\ln(7)$

$= 7(1 - \ln(7))$

≈ -6.62

So $y - (7 - 7\ln 7) = 0(x - \ln 7)$

or $y = 7 - 7\ln 7$

≈ -6.62

Plug in (+.5)

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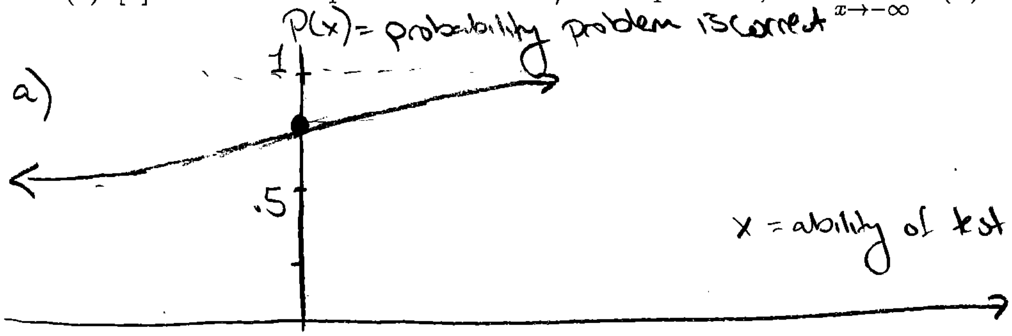
7. (WordProblems#1) Test makers use item response functions $P(x)$ to determine the difficulty and effectiveness of a given test question. The variable x is the ability of a test taker and $P(x)$ is the probability that the test taker gets the problem correct. By convention we let an "average ability" correspond with $x = 0$. Thus $P(0) = .75$ means that a person with average ability has a 75% chance of getting the question correct.

   (a) [2] Assume we have a well constructed True/False question. Sketch a possible response function $P(x)$ so that $P(0) = .75$. Note that you do not need to put units on the x axis but should have units on the vertical axis.

   (b) [2] On a well constructed question, what do we expect $\lim_{x\to\infty} P(x)$ to equal? Justify your answer.

   (c) [2] Assume the question is a True/False question, find $\lim_{x\to-\infty} P(x)$. Justify yourself.

a)



(+.5) start
(+.5) values on y axis
(+.5) plot (0, .75)
(+.5) continuous/ domain/range

b) We expect $\lim_{x\to\infty} P(x) = 1$ (+.5) sense (+.5)

That is, as the ability of a test taker increases, the probability of the test taker being correct should approach 100%. (+1)

c) We expect $\lim_{x\to-\infty} p(x) = \frac{1}{2}$ (+.5) sense (+.5)

As the ability of a test taker decreases the probability of the answer given being correct should be the same as random chance - or one out of 2 answers, so $\frac{1}{2}$ (+1)

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