TMATH 124: Quiz 4 Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. No calculators or notes are allowed.

1. [3] (LogWks #2) Find $\frac{dy}{dx}$ when $y = \log_2[(x^3 - x^2)\cos(x)]$

$y = \log_2[(x^3-x^2)\cos x]$

log property (+.5): $= \log_2(x^3-x^2) + \log_2(\cos x)$

$\frac{d}{dx}\Big($ $\frac{1}{(x^3-x^2)\ln 2}(3x^2-2x) + \frac{1}{(\cos x)\ln 2}\cdot(-\sin x)$

$\frac{1}{(x^3-x^2)\ln 2}$ (+.5), $\ln 2$ (+.5), chain (+.5); $\frac{1}{(\cos x)\ln 2}$ (+.5)

$y = \log_2[(x^3-x^2)\cos(x)]$

or $y' = \frac{1}{(x^3-x^2)(\cos x)\ln 2}[(x^3-x^2)\cos x]'$ (+1.5); chain (+.5)

$= \frac{1}{(x^3-x^2)(\cos x)(\ln 2)}[(x^3-x^2)(-\sin x) + (3x^2-2x)\cos x]$ (+.5)

product (+.5)

2. [3] (§3.6 #38) Let $f(x) = \log_a(3x^2 - 2)$. Find a so that $f'(1) = 3$.

$f(x) = \log_a(3x^2-2)$ chain (+.5)

$\Rightarrow f'(x) = \frac{1}{(3x^2-2)\ln a}(6x-0)$ (+.5), (+.5)

So if $f'(1) = 3$

(+.5) $\frac{1}{(3(1)^2-2)\ln a}(6(1)-0) = 3$

$\Rightarrow \frac{6}{\ln a} = 3$

$\frac{6}{\ln a} = 3\ln a$

$\frac{6}{3} = \frac{3\ln a}{3}$

$2 = \ln a$

$e^2 = e^{\ln a}$

$e^2 = a$

alg to solve for a (+1)

3. [4] (WebHW12 #7) Find the linearization of the curve $y = \ln(x^2 - 5x + 1)$ at $x = 5$.

(+.5) i.e. find the line tangent to $y = \ln(x^2-5x+1)$ when $x=5$

Looking for $y = mx+b$ or $y - y_1 = m(x-x_1)$

m = slope of line tangent to y when $x=5$

$= y'(5)$

$= \frac{2(5)-5}{5^2-5\cdot5+1}$ (1.5)

$= \frac{10-5}{1} = 5$

finding y':

$y' = \frac{1}{(x^2-5x+1)\ln e} \cdot (2x-5)$

$= \frac{2x-5}{x^2-5x+1}$ (+1)

(+1) passes thru $(5, \ln(5^2-5\cdot5+1)) = (5, \ln(1)) = (5, 0)$

so $y - 0 = 5(x-5)$ or $0 = 5(5)+b$

$\Rightarrow b = -25$

plug in (1.5)

so $y = 5x - 25$

Notation (+.5)