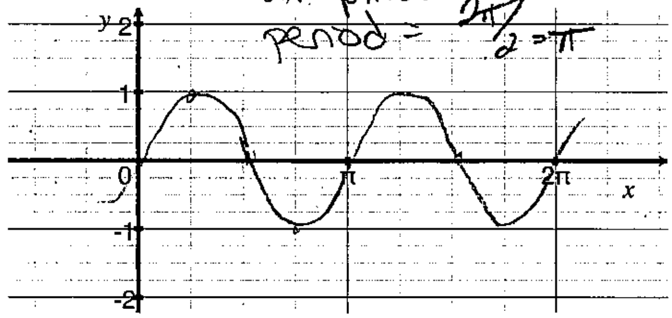
TMATH 124: Quiz 3

Key

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. No calculators or notes are allowed.

Amplitude = 1
period = $2\pi/2 = \pi$

1. Consider the graph of $f(x) = \sin(2x)$.

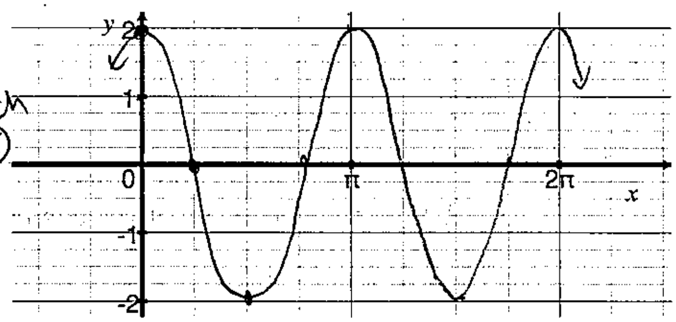


   (a) [1] Carefully, draw the graph of $f(x)$ on the axis provided.

   shape (+.5)
   got it (+.5)

   (b) [1] Sketch the graph of $\frac{df}{dx}$ on the axis below.

   shape idea (+.5)
   got it (+.5)



   (c) [2] (WebHW9 #7) Find $f'(x)$

   (+.5) { inside function $2x$ / outside function $\sin x$

   der of each (+.5)

   (+.5) Chain

   (+.5) $2\cos(2x)$

   (d) [2] (Trig Wks #2)
   Find $\lim\limits_{x\to 0} \frac{f(x)}{6x}$ (+.5)

$$= \lim_{x\to 0} \frac{\sin 2x}{6x} = \lim_{x\to 0} \frac{1 \cdot \sin 2x}{3 \cdot 2x}$$

$$= \lim_{x\to 0} \frac{1}{3} \cdot \lim_{x\to 0} \frac{\sin \cancel{2x}}{\cancel{2x}}$$

1 (+.5)

algebra (+.5)

limit eval/prop (+.5)

$$= \frac{1}{3}$$

2. (§3.2 #44) Given that $f(2) = -3$, $f'(2) = -2$, $g(2) = 4$, and $g'(2) = 7$, find the following:

(a) [2] $\frac{d}{dx}\left(\frac{1+f(x)}{g(x)}\right)\Big|_{x=2}$

quotient rule (+.5)

$$= \frac{g(x)\frac{d}{dx}(1+f(x)) - (1+f(x))\frac{d}{dx}(g(x))}{(g(x))^2}\Bigg|_{x=2}$$

(+.5)

$$= \frac{g(x)\frac{df}{dx} - (1+f(x))\frac{dg}{dx}}{(g(x))^2}\Bigg|_{x=2}$$

plug in (+1)

$$= \frac{g(2)f'(2) - (1+f(2))g'(2)}{(g(2))^2}$$

$$= \frac{4\cdot(-2) - (1+-3)7}{4^2}$$

$$\frac{-8-(-2)\cdot 7}{16} = \frac{-8+14}{16} = \frac{6}{16} = \frac{3}{8}$$

(b) [2] Find the equation of the line tangent to g when $x = 2$

(+.5) Looking for $y = mx+b$ or $y - y_1 = m(x - x_1)$

(+.5) m = slope of line tangent to g at $x = 2$

$= g'(2)$

$= 7$

(+.5) passes thru the point $(2, g(2))$ or $(2, 4)$

plug in (+.5)

So

$y - 4 = 7(x-2)$

or

$4 = 7(2) + b$

$4 = 14 + b$

$-14 \quad -14$

$-10 = b$

so $y = 7x - 10$