Key

TMATH 124: Quiz 2

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work. No calculators or notes are allowed.

1. [2] (WebHW4 #10) Find $\lim_{x\to\infty} \frac{1-9x}{2x+7}$

Table

x	10000

precalc method

$\lim_{x\to\infty} \frac{-9x+1}{2x+7} = -9/2$

got it (+.5)
leading terms (+1)
notation (+.5)

Ploy's Method

$\lim_{x\to\infty} \frac{1-9x}{2x+7}\frac{(1/x)}{(1/x)}$

or

$= \lim_{x\to\infty} \frac{1/x - 9x/x}{2x/x + 7/x}$

$= \lim_{x\to\infty} \frac{\overset{\to 0}{1/x} - 9}{2 + 7/x}$

$= -9/2$

notation (+.5) factor of 1/x (+.5) alg (+.5) big-Limit (+.5)

or

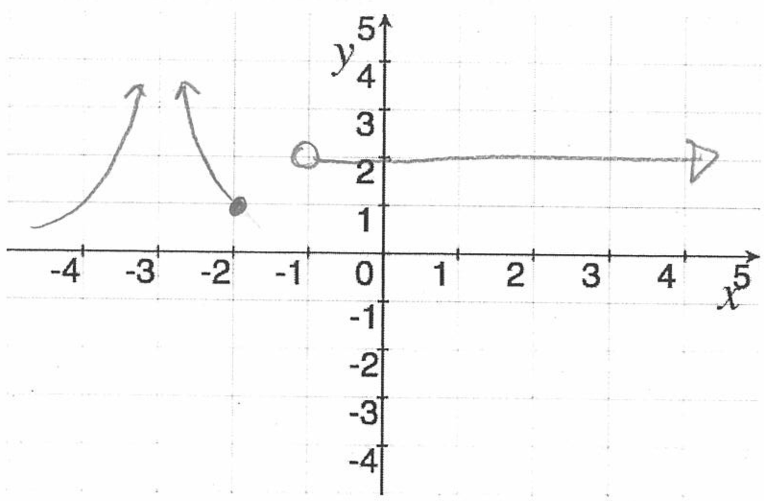
large x's (+.5)
notation (+.5)
computations (+.5)
got it (+.5)

2. [3] (Con't wks #6) Draw a function g such that both conditions are met: (+.5)

(a) $\lim_{x\to -3} g(x) = \infty$ (+1)

(b) $\lim_{x\to\infty} g(x) = 2$ (+1)

(c) g is continuous $-1 < x \leq 2$ (+.5)



note: there are MANY correct answers!

we could write the rule for this

$$g(x) = \begin{cases} \frac{1}{(x+3)^2} & \text{if } x \leq -2 \\ 2 & \text{if } -1 < x \end{cases}$$

3. (§2.7 #8) Let $f(x) = \dfrac{3x+1}{x+1}$.

(a) [3] Find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \underbrace{\frac{f(x+h)-f(x)}{h}}_{+.5} = \lim_{h\to 0} \frac{\overbrace{\frac{3(x+h)+1}{x+h+1}}^{+.5} - \overbrace{\frac{3x+1}{x+1}}^{+.5}}{h}$$

fractions (+.5)

alg (+.5)

$$= \lim_{h\to 0} \frac{(3x+3h+1)(x+1) - (3x+1)(x+h+1)}{(x+1)(x+h+1)} \div h$$

$$= \lim_{h\to 0} \frac{(\cancel{3x^2}+\cancel{3x}+\cancel{3xh}+3h+x+\cancel{1}) - (\cancel{3x^2}+\cancel{3x}h+\cancel{3x}+x+h+\cancel{1})}{(x+1)(x+h+1)} \div \frac{h}{1}$$

$$= \lim_{h\to 0} \frac{3h-h}{(x+1)(x+h+1)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(x+1)(x+h+1)}$$

$$= \underbrace{\lim_{h\to 0} \frac{2}{(x+1)(x+h+1)} = \frac{2}{(x+1)(x+0+1)}}_{+.5} = \frac{2}{(x+1)(x+1)} = \frac{2}{(x+1)^2}$$

(b) [2] Find the equation of the line tangent to f when $x = 1$

(+.5) { Looking for $y = mx+b$ or $y - y_1 = m(x - x_1)$

(+.5) { m = slope of line tangent to f at $x = 1$

$$= f'(1)$$

$$= \frac{2}{(1+1)^2} = \frac{2}{2^2} = \frac{2}{4} = \frac{1}{2}$$

(+.5) { passes thru the point $(1, f(1)) = \left(1, \frac{3\cdot 1+1}{1+1}\right) = (1, 2)$

(+.5) { So $y - 2 = \frac{1}{2}(x-1)$

or

$$2 = \tfrac{1}{2}(1) + b$$
$$2 - \tfrac{1}{2} = b$$
$$\tfrac{3}{2} = b$$

So $y = \frac{1}{2}x + \frac{3}{2}$