error 3b+6

Key

EXAM 2 — TMath 124 — Spring 2015

1. [6] TRUE/FALSE: Let f and g be functions. Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $x + \frac{1}{x} = \frac{x^2+1}{x}$ $\qquad \frac{x}{x}\frac{x}{1} + \frac{1}{x} = \frac{x^2+1}{x}$

T F $\sqrt{x} = x^{\frac{1}{2}}$

T F If $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist. $\qquad$ Consider $\lim_{x\to 0} \frac{\sin x}{x}$

T F If f and g are differentiable functions where $f'(1) = 2$, and $g'(1) = 3$, then $(fg)'(1) = 6$. $\qquad (f\cdot g)'(1) = f(1)g'(1) + f'(1)g(1) = ?\,3 + 2\cdot ?$ $\qquad (f\cdot g)' \neq f'g'$

may not be ?

T F If f is differentiable at a, then f is continuous at a.

T F $(2^x)' = 2^x$. $\qquad (2^x)' = 2^x(\ln 2)$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

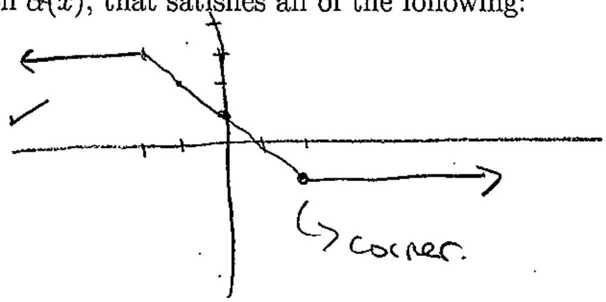
2. [5] (PracticeExam #3) Write a function $\alpha(x)$, that satisfies all of the following: (+1)

(+1) (a) $\lim_{x\to -\infty} \alpha(x) = 3$, ✓

(+1) (b) α is not differentiable at $x = 2$, ✓

(+1) (c) α is continuous at $x = 2$, ✓

(+1) (d) $\alpha'(0)$ is negative ✓



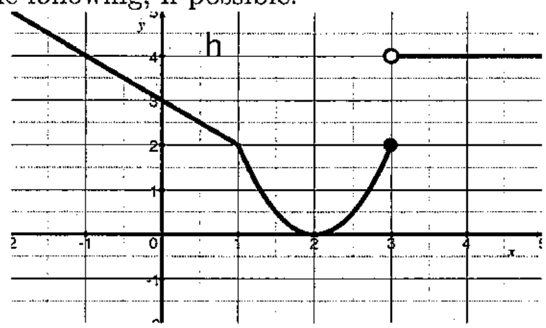
Note: there are many correct answers?

up to +2 for graph

$$\alpha(x) = \begin{cases} 3 & \text{if } x \le -2 \\ -x+1 & \text{if } -2 \le x \le 2 \\ -1 & \text{if } 2 \le x \end{cases}$$

3. Let h be the piece-wise defined function comprised of two line segments and a parabola shifted horizontally shown below and to the right. Let f be a continuous function with the characteristics described below. Find the following, if possible.

$f(-1) = -3$
$f(4) = 5$
$f'(-1) = -2$
$f'(4) = 1$



(a) [1] $\frac{d}{dx}(h(x))\,|_{x=0}$

(+.5) slope of line tangent to h at x=0

-1

(+.5)

[3] PracticeExam #5

$(f \circ h)'(-1)$

Chain Rule (+.5)

chain right (+.5)

$f'(h(-1)) \cdot h'(-1)$

$f'(4)\,(-1) = 1 \cdot (-1) = -1$

(+.5) (+.5) (+.5) notation (+.5)

[3] (Product Wks #1)

$(fh)'(-1)$

Product Rule (+.5)

product right (+.5)

$f(-1)\,h'(-1) + f'(-1)\,h(-1)$

(+.5) (+.5) (+.5) (+.5)

$(-3)(-1) + (-2)(4)$

$3 - 8 = -5$

[3] (§3.2 #44)

$$\frac{d}{dx}\left(\frac{f(x)}{3+h(x)}\right)\Bigg|_{x=-1}$$

quotient Rule (+.5)

quotient right (+.5)

$$\frac{[3+h(-1)]f'(-1) - f(-1)\,[3+h(\ )]'}{[3+h(-1)]^2}$$

(+.5) (+.5)

$$\frac{(3+4)(-2) - (-3)\,h'(-1)}{(3+4)^2} = \frac{-14 + 3\cdot(-1)}{7^2}$$

$= \frac{-17}{49}$

(b) [3] (Quiz3 #2) The linearization of f at $x = \cancel{2}$ $x = -1$

(+.5) i.e. find the equation of the line tangent to f @ x=-1

(+.5) Looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$

m = slope of line tangent to graph of f at x = -1

(+.5) $= f'(-1)$

(+.5) -2

(+.5) line passes thru $(-1, f(-1))$ or $(-1, -3)$

So

(+.5) $y + 3 = -2(x+1)$

$-3 = -2(-1) + b$

$\rightsquigarrow b = -5$

or $y = -2x - 5$

4. Find each of the following.

[2] (TrigWks #2)

$$\lim_{x\to 0} \frac{x^2}{\sin(2x)\sin(5x)}$$

$\lim_{x\to 0} \frac{x}{\sin(2x)} \cdot \frac{x}{\sin(5x)}$ (+.5) Recall $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

$= \lim_{x\to 0} \frac{1}{2} \frac{2x}{\sin 2x} \cdot \frac{1}{5} \cdot \frac{5x}{\sin 5x}$ multipliers (+.5)

$= \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$ dg (+1)

or

started table (+.5) double sided (+.5) repeated answer (+1)

x	$-.1$	$-.00001$	$.00001$	$.01$
$\frac{x^2}{\sin(2x)\sin(5x)}$				

[2] (WebHW9 #8)

$$\frac{d}{dx}\left(6^{15x}\right)$$

Chain Rule (+.5)

(+.5) $f(x) = 6^x$, $g(x) = 15x$ (+.5) $f'(x) = 6^x \ln 6$, $g'(x) = 15$

$f'(g(x))g'(x) = f'(15x) \cdot 15$

$= 6^{15x} \ln 6 \cdot 15$ (+.5)

[4] (implicit wks #1)

$$(\sin^5(x)\sqrt{x^3-5})'$$

product rule (+1)

$\sin^5(x)\left[(x^3-5)^{1/2}\right]' + \left[\sin^5(x)\right]'\sqrt{x^3-5}$

chain rule (+1) chain rule (+1)

$\sin^5(x)\ \frac{1}{2}(x^3-5)^{-1/2} \cdot 3x^2 + 5\sin^4(x)\cos(x)\sqrt{x^3-5}$

pieced together (+1)

[4] (PracticeExam #6)

$$\frac{d}{dy}\left(\frac{\sin(y)+y\cos(y)}{\cos(y)}\right) = \frac{d}{dy}\left(\frac{\sin y}{\cos y} + \frac{y\cancel{\cos y}}{\cancel{\cos y}}\right)$$

simplify (+1)

$= \frac{d}{dy}(\tan y + y)$

$= \sec^2 y + 1$ wrt y (+1)

(+1) (+1)

or

wrt y (+1)

$$\frac{\cos(y)\left[\cos(y) + y(-\sin(y)) + (1)\cos y\right] - (\sin(y)+y\cos(y))(-\sin y)}{\cos^2 y}$$

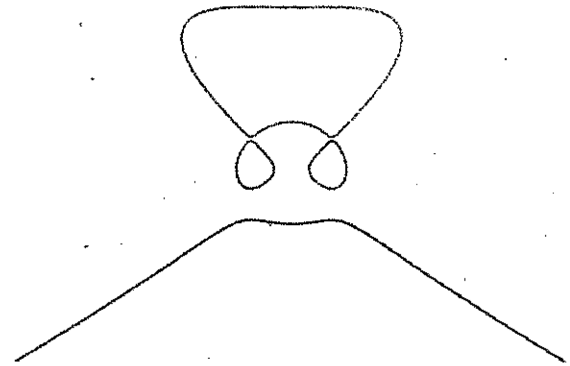
$\frac{d}{dy}\sin(y)$ (+.5)

$\frac{d}{dy}\cos y$ (+.5)

quotient (+1)

product (+1)

5. The graph of the equation
$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$
without the the axis has been described as a bouncing wagon.



(a) [4] (implicit Wks #1) Find $\frac{dy}{dx}$ as a function of x and y of the bouncing wagon shown to the right.

notation (+.5)

$\frac{d}{dx}(2y^3 + y^2 - y^5) = \frac{d}{dx}(x^4 - 2x^3 + x^2)$

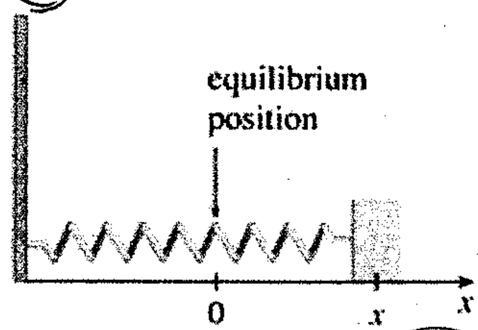
(+.5) (+.5) (+.5) (+1)

$6y^2 \frac{dy}{dx} + 2y \frac{dy}{dx} - 5y^4 \frac{dy}{dx} = 4x^3 - 6x^2 + 2x$

$\frac{dy}{dx}(6y^2 + 2y - 5y^4) = 4x^3 - 6x^2 + 2x$

$\frac{dy}{dx} = \dfrac{4x^3 - 6x^2 + 2x}{6y^2 + 2y - 5y^4}$ algebra (+1)

6. (WebHW9 #6) A mass on a spring vibrates horizontally on a smooth level surface with the equation $x(t) = 10\cos(2t)$ where t is in seconds and x is in centimeters.



(a) [2] Find the velocity of the spring at time t.

(+.5) velocity $= \frac{d}{dt}$(position) Chain rule (+.5)

$= \frac{d}{dt}(10\cos(2t)) = 10(-\sin(2t)) \cdot 2 = -20\sin(2t)$

(+.5) alg/notation (+.5)

(b) [3] When is the spring at rest?

(+.5) when the velocity = 0

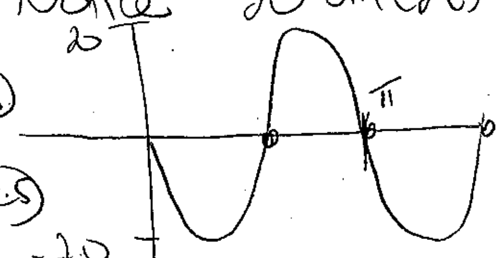
(+.5) from part (a) $0 = -20\sin(2t)$

one answer (+.5) Notice $-20\sin(2t)$ has the graph

more answers (+.5)

got all answers (+.5)

notation (+.5)



$\Rightarrow$ zeros every half multiple of π

— or —

$K\frac{1}{2}\pi$ where K is an integer

when $0 = \sin(2t)$

when

or $2t$ = multiples of π

so every multiple of half π

7. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§3.9 #20) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher then the bow of the boat. The rope is being pulled in at a rate of 2m/s.

i. [3] Find an equation relating the speed of the boat to other variables.

ii. [2] How fast is the boat approaching the dock when it is 5 m from the dock?

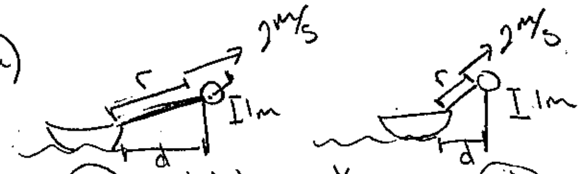
(b) (WordWks2 #12) Consider a ladder 12ft long leaning against a vertical wall where the bottom of the ladder is sliding away front the wall at a rate of 1ft/s.

i. [3] Find an equation relating the speed that the angle between the ladder and the ground to other variables.

ii. [2] how fast is the angle between the latter and the ground changing when the bottom of the latter is 6ft from the wall?

(a)

i)



Start (+.5) label unknowns (+.5)

(+.5) WANT dd/dt HAVE $dr/dt = 2$ m/s

(+.5) find a relation between d and r

$d^2 + 1^2 = r^2$

(+.5) Take d/dt of both sides

(+.5) $2d \, dd/dt + 0 = 2r \, dr/dt$

or

$dd/dt = \dfrac{\cancel{2}r\,dr/dt}{\cancel{2}d} = \dfrac{r \cdot 2}{d}$

ii) WANT $dd/dt\big|_{d=5}$ (+.5)

(+.5)

$1^2 + 5^2 = r^2$

$\Rightarrow r = \sqrt{26}$

$dd/dt\big|_{d=5} = \dfrac{r \cdot 2}{5}$

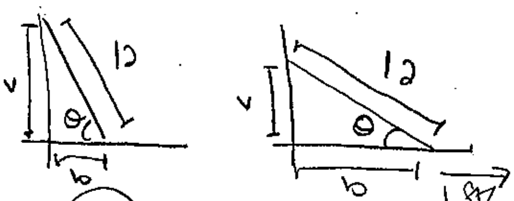
we need to know r when d = 5 (+.5)

So $dd/dt\big|_{d=5} = \dfrac{\sqrt{26} \cdot 2}{5}$ m/s (+.5) 5

≈ 2.0396 m/s

(b)

i)



Start (+.5) label unknowns (+.5)

(+.5) WANT $d\theta/dt$ HAVE $db/dt = 1$

(+.5) find a relation between θ & b

Sohcahtoa $\cos\theta = b/12$

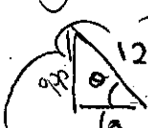
(+.5) Take d/dt of both sides

(+.5) $(-\sin\theta)\, d\theta/dt = \frac{1}{12}\, db/dt$

or $d\theta/dt = \frac{-1}{12} \frac{db}{dt} \frac{1}{\sin\theta}$

ii) WANT $d\theta/dt\big|_{b=6}$ (+.5)

$\dfrac{d\theta}{dt} = \dfrac{-1}{12}(1)\dfrac{1}{\sin\theta}$



opp = 6

$\sin\theta = \dfrac{opp}{12} = \dfrac{\sqrt{12^2 - 6^2}}{12}$

(+.5) we need to know $\sin\theta$ when b = 6

(+.5) So $\dfrac{d\theta}{dt}\Big|_{b=6} = \dfrac{-1}{\cancel{12}} \cdot \dfrac{\cancel{12}}{\sqrt{12^2 - 6^2}} = \dfrac{-1}{6\sqrt{3}}$