More Differentiation Practice

For each of the functions below find their respective derivatives.

1. $\sin(x^3 - 5)$ $\qquad (x^3-1)^{100} \qquad 5^{3x^2-x}$

Chain Rule

$g(x) = x^3 - 5 \quad g'(x) = 3x^2$

$f(u) = \sin u \quad f'(u) = \cos u$

$(\sin(x^3-5))' = f'(g(x))g'(x)$

$= f'(x^3-5) \cdot 3x^2$

$= \cos(x^3-5) \cdot 3x^2$

Chain Rule

$g(x) = x^3 - 1 \quad g'(x) = 3x^2$

$f(u) = u^{100} \quad f'(u) = 100u^{99}$

$[(x^3-1)^{100}]' = f'(g(x))g'(x)$

$= f'(x^3-1) \cdot 3x^2$

$= 100(x^3-1)^{99} \cdot 3x^2$

Chain Rule

$g(x) = 3x^2 - x \quad g'(x) = 6x-1$

$f(u) = 5^u \quad f'(u) = 5^u \ln 5$

$[5^{3x^2-x}]' = f'(g(x))g'(x)$

$= f'(3x^2-x) \cdot [6x-1]$

$= 5^{3x^2-x} \cdot \ln 5 \cdot [6x-1]$

2. Recall that we can use the product, quotient, and chain rule together! The trick is to use the notation to *guide* you. Find the derivative of $\sin^5(x)\sqrt{x^3-5}$.

$[\sin^5(x)\sqrt{x^3-5}]' = \sin^5(x)\underbrace{[\sqrt{x^3-5}]'}_{*} - \underbrace{[\sin^5(x)]'}_{\dagger}\sqrt{x^3-5}$ (by product rule)

$= (\sin^5 x)\frac{1}{2}(x^3-5)^{-1/2} \cdot 3x^2 - 5\sin^4 x \cos x \sqrt{x^3-5}$

Chain Rule:

$* \ [\sqrt{x^3-5}]' = [(x^3-5)^{1/2}]' = f'(g(x))g'(x) = f'(x^3-5) \cdot 3x^2$

$g(x) = x^3 - 5 \quad g'(x) = 3x^2$

$f(u) = u^{1/2} \quad f'(u) = \frac{1}{2}u^{-1/2}$

$= \frac{1}{2}(x^3-5)^{-1/2} \cdot 3x^2$

$[\sin^5 x]' = [(\sin x)^5]' = 5\sin^4 x \cdot \cos x$

$g(x) = \sin x \quad g'(x) = \cos x$

$f(u) = u^5 \quad f'(u) = 5u^4$

3. The chain rule can also be used in conjunction with itself. That is, we can use the chain rule to work on a derivative, but when trying to find the "inside function", we may need to use the chain rule *again*. Find the derivative of $\sin^2(x^3)$.

$[\sin^2(x^3)]' = [(\sin(x^3))^2]' = f'(g(x))g'(x) = f'(\sin(x^3)) \cdot \cos(x^3) \cdot 3x^2$

$= 2\sin(x^3) \cdot \cos(x^3) \cdot 3x^2$

$g(x) = \sin(x^3) \quad g'(x) = \underbrace{\cos(x^3) \cdot 3x^2}_{*}$

$f(u) = u^2 \quad f'(u) = 2u$

$* \ g'(x) = [\sin(x^3)]' = f'(g(x)) \cdot g'(x) = f'(x^3) \cdot 3x^2$

$= \cos(x^3) \cdot 3x^2$

$g(x) = x^3 \quad g'(x) = 3x^2$

$f(u) = \sin u \quad f'(u) = \cos u$