TMATH 124 Quiz 3

Key.

Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. [4] Find the following:

(product & quotient wks #3)

$$\left(\frac{6x^2-\sqrt{x}}{2x}\right)' = \left[\frac{6x^2}{2x} - \frac{x^{1/2}}{2x}\right]'$$

alg +1, power rule +1, add/const +1

$$= \left[3x - \tfrac{1}{2}x^{-1/2}\right]' = [3x]' - \left[\tfrac{1}{2}x^{-1/2}\right]'$$

$$= 3 - \tfrac{1}{2}\left[x^{-1/2}\right]' = 3 - \tfrac{1}{2}\left(-\tfrac{1}{2}\right)x^{-3/2}$$

$$= 3 + \tfrac{1}{4}x^{-3/2}$$

or

$$\frac{2x\left[6x^2-\sqrt{x}\right]' - \left(6x^2-\sqrt{x}\right)(2x)'}{(2x)^2}$$

quotient +1, rgt quotient +1.5

$$= \frac{2x\left(\left[6x^2\right]' - \left[x^{1/2}\right]'\right) - \left(6x^2-\sqrt{x}\right)\cdot 2}{4x^2}$$

+1.5

$$= \frac{2x\left(12x - \tfrac{1}{2}x^{-1/2}\right) - 2\left(6x^2-\sqrt{x}\right)}{4x^2}$$

+1.5

(§3.4 #24)

$$\frac{d}{dx}\left(10^{3-x^2}\right)$$

$g(x) = 3-x^2 \quad g'(x) = -2x$

$f(u) = 10^u \quad f'(u) = (\ln 10)10^u$ +1.5

$$= f'(g(x))\,g'(x)$$

$$= f'(3-x^2)\cdot(-2x)$$

$$= 10^{3-x^2}(\ln 10)(-2x)$$

chain +1.5, try 1.5, setup right 1

2. [2] (trig wks #2) Determine the following, if it exists. Be sure to justify your work.

$$\lim_{x\to 0}\frac{x\cos\left(x+\frac{\pi}{4}\right)}{\sin(x\sqrt{2})} = \lim_{x\to 0}\frac{x}{\sin(x\sqrt{2})}\cdot\frac{\cos(x+\pi/4)}{1}\cdot\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

factor +1.5, rgt one +1.5

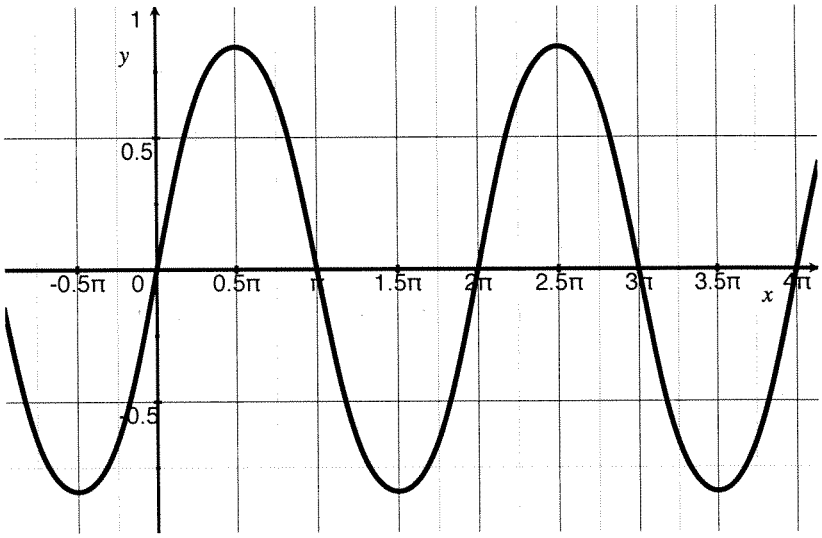
$$= \lim_{x\to 0}\frac{x\sqrt{2}}{\sin(x\sqrt{2})}\cdot\frac{\cos(x+\pi/4)}{\sqrt{2}} = \lim_{x\to 0}\frac{\cos(x+\pi/4)}{\sqrt{2}}$$

$$= \frac{\cos(0+\pi/4)}{\sqrt{2}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\div\frac{\sqrt{2}}{1} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2}$$

limits 1.5, alg/trig 1.5

3. Consider the function $f(x) = \sin(\sin(x))$ graphed below.

(a) [3] (WebHW9 #10) Find the equation of the line tangent to the curve below when $x = \pi$.



$y = mx + b$ (+.5)

m = slope of line tangent to f when $x = \pi$ (+.5)

$= f'(\pi)$

(+.5) $f'(x) = [\sin(\sin(x))]'$

inside function $\sin x$ inside' $\cos x$

outside function $\sin u$ outside' $\cos u$

$= \cos(\sin(x)) \cdot \cos(x)$

$\Rightarrow f'(\pi) = \cos(\sin(\pi))\cos(\pi)$ (+.5)

$= \cos(0)\cos(\pi)$

$= 1(-1) = -1$

So $m = -1$

(+1) The line passes thru $(\pi, f(\pi)) = (\pi, \sin(\sin\pi)) = (\pi, 0)$

So $0 = -1\pi + b \Rightarrow b = \pi$

Thus $y = -x + \pi$

(b) [1] Find all x values with the property that $f'(x) = 0$.

from the graph it looks like $\pm\frac{1}{2}\pi, \pm\frac{3}{2}\pi, \pm\frac{5}{2}\pi$, etc (+.5)

ie everywhere the line tangent to f is a horizontal line (+.5)