TMATH 124 Quiz 2

Key

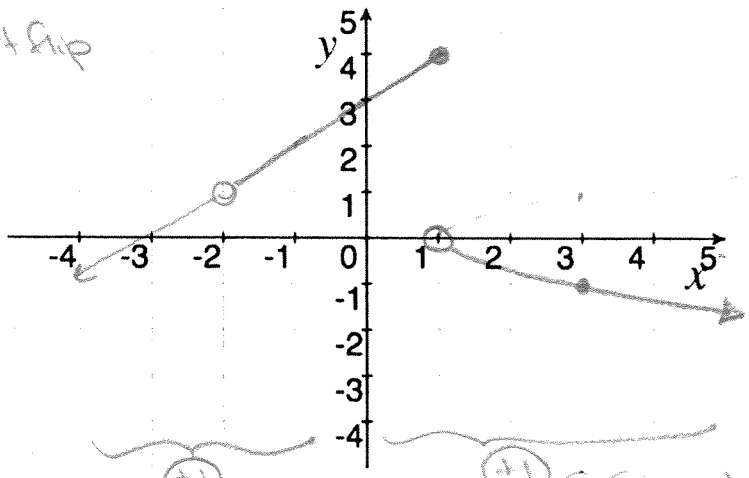
Show *all* your work (numerically, algebraically, or geometrically) for each and simplify. No credit is given without supporting work.

1. (con't wks #1) Consider the piecewise-defined function f defined below:

$$f(x) = \begin{cases} \dfrac{(x+2)(x+3)}{x+2} & \text{if } x \le 1 \\ -\log_3 x & \text{if } 1 < x \end{cases}$$

vert flip

(a) [2] Draw the graph of f on the axis provided.



(b) [2] Where is f continuous? Explain why.

(+1) { everywhere but when $x=1$ and $x=-2$

(+1) { b/c $f(1) = 4 \neq 0 = \lim_{x\to 1^+} f(x)$

(+1) $f(-2)$ DNE $\neq \lim_{x\to -2} f(x)$

I have to pick up my pencil in 2 places when drawing the function

2. [3] (WebHW5 #2 & inf limit wks) Determine the following, if they exist. Be sure to justify your work.

$$\lim_{x\to\infty} \frac{8x-5}{2x+6}$$

$\frac{1}{x}$ $\frac{1}{x}$

$$= \lim_{x\to\infty} \frac{\frac{8x}{x} - \frac{5}{x}}{\frac{2x}{x} + \frac{6}{x}}$$

$$= \lim_{x\to\infty} \frac{8 - 5/x}{2 + 6/x}$$

$$= \frac{\lim_{x\to\infty} 8 - \lim_{x\to\infty} 5/x \to 0}{\lim_{x\to\infty} 2 + \lim_{x\to\infty} 6/x \to 0}$$

$= 8/2 = 4$

result (+.5)
notation (+.5)
approach (+.5)
alg (+.5)

$$\lim_{x\to\infty} (x - x^2) = \lim_{x\to\infty} x(1-x)$$

"Big" · "neg big" = neg big

$-\infty$

notation/sense (+.5)
approach (+.5)

3. (§2.7 #3) Consider the function $f(x) = 6x - x^2$.

(a) [2] Find the slope of the line tangent to f when $x = 1$.

slope of line tangent to f when $x=1$ $= f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{[6(1+h)-(1+h)^2] - [6\cdot 1 - 1^2]}{h}$ (+1)

$= \lim_{h\to 0} \frac{6+6h-1-2h-h^2-5}{h} = \lim_{h\to 0} \frac{4h-h^2}{h}$

$= \lim_{h\to 0} \frac{h(4-h)}{h} = \lim_{h\to 0} (4-h) = 4-0 = 4$

alg (+1,5) notation (+1,5)

(4)

(b) [1] Find the equation of the line tangent to f when $x = 1$.

Looking for $y = mx+b$

from (a) we know $m=4$,

(+.5) { Line passes thru $(1, f(1)) = (1, 6\cdot 1 - 1^2) = (1, 5)$

(+.5) { so $5 = 4(1) + b$

$\Rightarrow b = 1$ $\Rightarrow y = 4x+1$

or

Looking for $y - y_1 = m(x - x_1)$

from (a) we know $m=4$

(+.5) { Line passes through $(1, f(1)) = (1, 6\cdot 1 - 1^2) = (1, 5)$

(+.5) { so $y - 5 = 4(x-1)$