FINAL

TMath 124

Practice

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be differentiable functions and h be a constant.

   T F $\frac{x+h}{2x} = \frac{1+h}{x}$

   T F $\sqrt{x^2+h^2} = x+h$

   T F $\lim_{x\to r} f(x) = f(r)$ for all r in the domain of f.

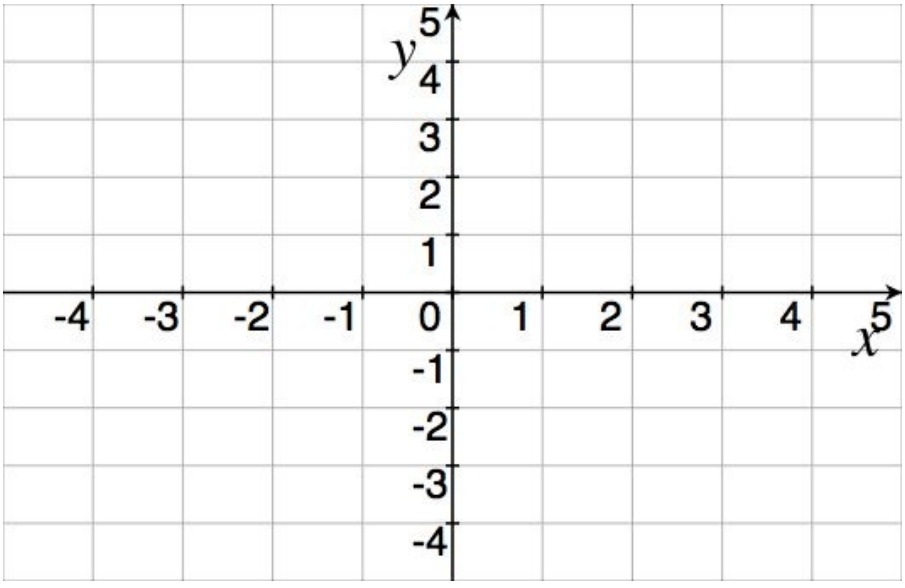
   T F If $\lim_{x\to r} g(x) = 0$, then $\lim_{x\to r} \frac{f(x)}{g(x)}$ does not exist.

   T F $\frac{d}{dx}(\frac{1}{x}) = -1$

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [] Sketch the graph and then *find the formula* of an example function f that satisfies the following conditions:

   (a) $f(2) = 2$
   (b) $\lim_{x\to 2} f(x) = -4$
   (c) f is not differentiable when $x = -3$
   (d) f is continuous when $x = -3$
   (e) $\lim_{x\to 0^+} f(x) = \infty$
   (f) $f'(4) = 2$



3. Compute the following limits:

(a) $\lim_{x\to 1} \dfrac{x^2+x-2}{2x^2-8x+6}$

(b) $\lim_{x\to \infty} \dfrac{x^2+x-2}{2x^2-8x+6}$

(c) $\lim_{\theta\to 0^+} \dfrac{\theta+\theta^2}{1-\cos\theta}$

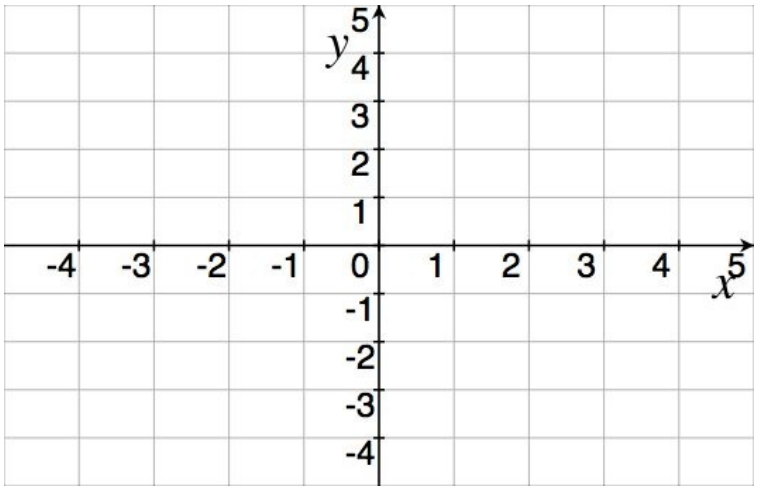
(d) $\lim_{x\to \infty} x\sin\left(\dfrac{5\pi}{x}\right)$

(e) $\lim_{x\to 0} x^4\sin\left(\dfrac{1}{x}\right)$

(f) $\lim_{x\to 1} \dfrac{1}{x-1}$

4. Let $f(x) = \begin{cases} \sqrt{1-(x+3)^2} & \text{if } -4 \le x \le -2 \\ 1 & \text{if } -2 < x < 1 \\ -(x-2)^2 + 2 & \text{if } 1 < x \end{cases}$

Graph $f(x)$ and then sketch the graph $f'(x)$ below on its own set of axes. Afterwards, answer the following questions.

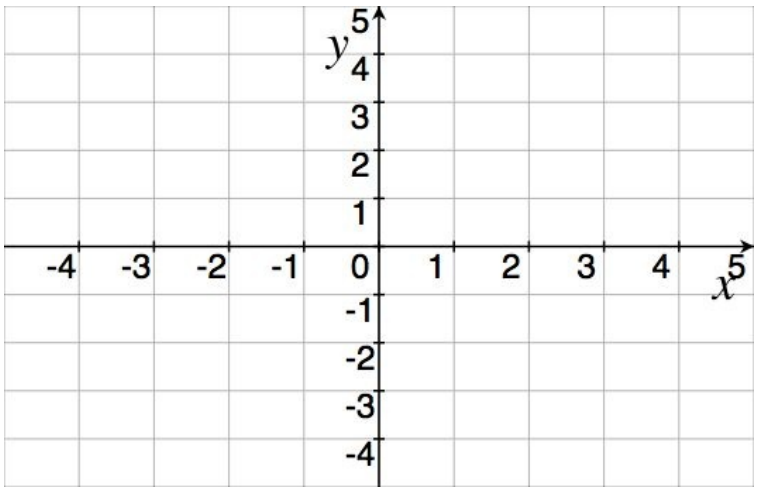


(a) $\lim_{x \to 1} f(x)$

(b) $\lim_{x \to 3} [4f(x) - 7]$

(c) $\lim_{x \to -2} f(x)$

(d) $\lim_{x \to -2^-} f(x)$



(e) $\lim_{x \to 3} f'(x)$

(f) $\lim_{x \to \infty} f(x)$

(g) $[f + f]'(2)$

5. Compute the derivatives of the following functions. You do *not* need to simplify.

(a) $f(x) = x^3 + 3^x + \pi^\pi$

(b) $g(t) = \ln(t)\left(\dfrac{2+t^2}{3t-1}\right)$

(c) $h(\theta) = 7\sec(\sqrt{\theta})$

(d) $y = \sqrt{x}e^{x^7}(x^6+3)^{10}$

(c) $y = (\cos(x))^x$

(d) $x^2y^2 = 4 - y\arctan(5x)$

6. Find the equation of the line tangent to the graph of f when $x = 2$ if $f(x) = m(n(x))$, $n(2) = -1$, $m(-1) = 6$, $n'(2) = 3$, and $m'(-1) = 5$.

7. Find the antiderivative for each of the following functions:

   (a) $2x - x^3 + 7\sin(x)$ (b) $\dfrac{5 - 4x^3 + 2x^6}{x^6}$

8. Consider the function $f(x) = \sqrt[3]{x}$

   (a) Evaluate the integral $\displaystyle\int_1^8 \sqrt[3]{x}\, dx$

   (b) Draw a picture that corresponds to the area you computed in (a).

9. A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2\text{m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3m deep.

10. When blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel: $F = kR^4$. A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow.

    Use a linear approximation to show that the relative change in F is about four times the relative change in R. Then approximate how a 5% increase in the radius will affect the flow of blood?

11. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 7 - x^2$

12. A truck has a minimum speed of 9 mph in high gear. When traveling x mph, the truck burns diesel fuel at the rate of

$$0.003935\left(\frac{675}{x} + x\right)\frac{\text{gal}}{\text{mile}}$$

Assume that the truck can not be driven over 63 mph, that diesel fuel costs \$2.84 a gallon, and that the driver is paid \$12 an hour. Find the speed that will minimize the cost of a 500 mile trip.