3. For each f defined below, find $f'(x)$.

$$f(x) = x^4 + 2e^x \qquad\qquad f(x) = e^{x+4} - 7e^2 \qquad\qquad f(x) = \frac{e^x + 7}{e}$$

$f'(x) = [x^4 + 2e^x]'$

$\quad = [x^4]' + [2e^x]'$

$\quad = 4x^3 + 2[e^x]'$

$\quad = 4x^3 + 2e^x$

$\frac{d}{dx} f(x) = \frac{d}{dx}[e^{x+4} - 7e^2]$

$\quad = \frac{d}{dx}[e^4 e^x - 7e^2]$

$\quad = \frac{d}{dx}[e^4 e^x] - \frac{d}{dx}[7e^2]$

$\quad = e^4 \frac{d}{dx}[e^x] - \frac{d}{dx}[7e^2 x^0]$

$\qquad\qquad\qquad\downarrow$ const

$\qquad\quad$ const

$\quad = e^4 e^x = 0 \qquad = e^{4+x}$

$f'(x) = \left[\frac{e^x + 7}{e}\right]'$

$\quad = [\frac{1}{e} e^x + \frac{7}{e}]'$

$\quad = [\frac{1}{e}e^x]' + [\frac{7}{e}x^0]'$

$\quad = \frac{1}{e}[e^x]' + \frac{7}{e}[x^0]'$

$\qquad\qquad\qquad\downarrow$ const

$\quad$ const

$\quad = \frac{1}{e}e^x + 0 = e^{x-1}$

4. Consider $\alpha(x) = x^4 + 2e^x$.

(a) Find the equation of the line tangent to the graph of α at the point $(0,2)$.

Looking for $y = mx + b$ or $y - y_1 = m(x - x_1)$

$m =$ slope of line $= \alpha'(0)$ from #3 we know $\alpha'(x) = 4x^3 + 2e^x$
tangent to α
when $x = 0$
$\qquad\qquad$ so $\alpha'(0) = 4 \cdot 0^3 + 2 \cdot e^0 = 0 + 2 \cdot 1 = 2$

$\Rightarrow m = 2$. The line passes thru $(0,2)$ so $2 = 2 \cdot (0) + b \Rightarrow b = 2$

$\Rightarrow \boxed{y = 2x + 2}$ or $\boxed{y - 2 = 2(x - 0)}$

(b) Find the line normal to the line you found in part (a) that also passes through the point $(0,2)$.

Recall two lines are normal if their slopes are perpendicular.
The line in (a) has slope 2 so a perpendicular line has slope $-\frac{1}{2}$.
The line passes thru $(0,2)$ so $2 = -\frac{1}{2}(0) + b \Rightarrow b = 2 \quad \Rightarrow \boxed{y = -\frac{1}{2}x + 2}$

5. At what point on the curve of $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?

Let $\beta(x) = 1 + 2e^x - 3x$. We want to find x so that

the slope of the line $\quad = \quad$ slope of $\quad\quad$ while $3x - y = 5$
tangent to β at x $\qquad\qquad 3x - y = 5 \qquad\qquad\qquad 3x - 5 = y$

$\quad\qquad -$ or $-$

$\beta'(x) = 3$ $\qquad\quad$ we can compute $\beta'(x) = [1 + 2e^x - 3x]'$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 2e^x - 3$

$2e^x - 3 = 3$ $\qquad e^x = \frac{6}{2}_2 \quad \ln e^x = \ln 3$

$2e^x = 6$ $\qquad\quad e^x = 3 \qquad \boxed{x = \ln 3}$