Explicit Differentiation Practice

For each of the functions below find their respective derivatives.

$47 + 3e^{3x^2-x}$

$[47+3e^{3x^2-x}]' = [47]' + [3e^{3x^2-x}]'$

$= 0 + 3[e^{3x^2-x}]' \underset{(\dagger)}{=} 3e^{3x^2-x}((6x-1)$

$(\dagger)$ $g(x) = 3x^2 - x$ $\quad g'(x) = 6x-1$

$f(u) = e^u$ $\quad f'(u) = e^u$

$f'(g(x))g'(x) = f'(3x^2-x)\cdot(6x-1)$

$= e^{3x^2-x}(6x-1)$

$\dfrac{x^2+7x}{\sqrt{x^3-5}} = \dfrac{x^2+7x}{(x^3-5)^{1/2}}$

$\left[\dfrac{x^2+7x}{(x^3-5)^{1/2}}\right]' = \dfrac{(x^3-5)^{1/2}[x^2+7x]' - (x^2+7x)\overbrace{[(x^3-5)^{1/2}]'}^{(*)}}{[(x^3-5)^{1/2}]^2}$

$= \dfrac{(x^3-5)(2x+7) - (x^2+7x)\frac{1}{2}(x^3-5)^{-1/2}3x^2}{(x^3-5)}$

$(*)$ $[(x^3-5)^{1/2}]' = f'(g(x))g'(x) = f'(x^3-5)\cdot 3x^2$

$= \frac{1}{2}(x^3-5)^{-1/2}\cdot 3x^2$

$g(x) = x^3-5$ $\quad g'(x) = 3x^2$

$f(u) = u^{1/2}$ $\quad f'(u) = \frac{1}{2}u^{-1/2}$

$\sin^5(x)\sqrt{x^3-5}$

$\sin^5 x\,[(x^3-5)^{1/2}]' + [\sin^5 x]'(x^3-5)^{1/2}$

$= \underbrace{(\sin^5 x)\tfrac{1}{2}(x^3-5)^{-1/2}(3x^2)}_{(*)} + \underbrace{5\sin^4 x\cos x}_{(!!)}(x^3-5)^{1/2}$

$(*)$ $[(x^3-5)^{1/2}]' = f'(g(x))g'(x) = f'(x^3-5)3x^2$

$g(x) = x^3-5$ $\quad g'(x) = 3x^2$ $\quad = \frac{1}{2}(x^3-5)^{-1/2}3x^2$

$f(u) = u^{1/2}$ $\quad f'(u) = \frac{1}{2}u^{-1/2}$

$(!!)$ $[\sin^5 x]' = [(\sin x)^5]' = f'(\sin x)\cdot\cos x$

$= 5\sin^4 x\cos x$

$g(x) = \sin x$ $\quad g'(x) = \cos x$

$f(u) = u^5$ $\quad f'(u) = 5u^4$

$\sqrt{47 + 3e^{3x^2-x}}$

$g(x) = 47 + 3e^{3x^2-x}$

$f(u) = u^{1/2}$

$f'(u) = \frac{1}{2}u^{-1/2}$

$g'(x)$ look at the 1st question (it's another chain rule)

$= e^{3x^2-x}(6x-1)$

$[\sqrt{47+3e^{3x^2-x}}]' = f'(g(x))g'(x)$

$= f'(47+3e^{3x^2-x})\cdot e^{3x^2-x}(6x-1)$

$= \frac{1}{2}(47+3e^{3x^2-x})^{-1/2}e^{3x^2-x}(6x-1)$